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The Quality–Quantity Trade-off

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The standard vertical-product-differentiation model assumes that quality and quantity are independent choices. That is, the choice of quality is separated from the decision of the number of units to produce. For certain products this is not appropriate. This note presents an extension to the vertical-product-differentiation model to incorporate the relationship between quality and quantity.
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INTRODUCTION

The purpose of this note is to extend the standard vertical-product-differentiation model to analyze the trade-off, often present, between the quality of a good and the quantity produced, and to study the implications of such a relationship. The standard model developed by Gabszewicz and Thisse [1979; 1980] and Shaked and Sutton [1982; 1983] implicitly assumes that quality and quantity are independent choices. That is, at any set level of quality, a firm is free to produce as much as it desires. Their models are set up as extensive-form games where the choice of quality is linked to quantity only, through the equilibrium refinement of subgame perfection, by the fact that for each possible choice of quality the firms will set their prices and quantities to maximize their profits.

This untested assumption can be rationalized. For example, the blueprints of a factory can be used to make a second factory identical to the first. Thus, quantity can be doubled with no change in the product being produced. While this may often be a reasonable assumption, in some markets it does not hold true. Consider a market for a good with a specialized or rare input. The use of the input cannot be expanded without an effect on the ability to maintain a certain level of quality. Consider, as an example, cigars. Cigars are a luxury good that do much more than just provide nicotine. Aficionados critique cigars on various dimensions where quality is discussed and enjoyed. For example, a popular magazine on cigars provides reviews by numerous professional reviewers. Reviewers, not observing price, brand, or origins of the tobacco, critique cigars based on esthetics, construction, flavor, and strength. A cigar's quality is determined by the environment in which the tobacco is grown and by the skill of the laborers. Furthermore, it is very costly to increase production. In fact, production can only be increased at the expense of lower quality. Making more cigars requires that leaves that would have otherwise been rejected (or used in lower-priced cigars) be used. Using large quantities of nitrogen fertilizer has the effect of growing more and larger tobacco leaves, but these leaves tend to be bitter and bad-tasting.[1] Also, rolling more cigars may require using less-skilled workers or machines, either of which would reduce the quality of the final product. Thus, there is a distinct link between the quantity and quality of the cigars produced. As another example, consider an instructor teaching a course. As the number of students enrolled increases, the demands on the instructor increase. The ability of the instructor to meet with students outside of class requires either significantly more time invested or a reduction in the time allotted per capita. Projects and examinations must be either eliminated or altered, or the instructor must incur a considerable cost. In both examples, there is an inverse relationship between quality and quantity; as one increases the cost to providing the other increases and, consequently, the amount is reduced. It is this feature of markets for such goods that is considered here.

In a similar setup, Sheshinski [1976] allows for a trade-off between quantity and quality. He considers a monopolist determining price, quantity, and quality to understand the distortions caused and the usefulness of regulation. He does not illustrate the effect of the quality–quantity trade-off and does not consider competition, which is done here.

In this note, I lay out a model of quality selection in an imperfectly competitive market, taking into account the trade-off between quality and quantity. I show that the stronger this relationship is, the more sales are shifted from the high-quality to the low-quality producer. Furthermore, a stronger relationship between quality and quantity results in the price of all goods being greater. These results extend and revise those of Gabszewicz and Thisse [1979; 1980] and Shaked and Sutton [1982; 1983] for goods that exhibit this trade-off.

THE MODEL

There are two ex ante identical firms, labeled A and B, and a continuum of consumers. Each consumer is assigned a type $\theta_i \in [\underline{\theta}, \bar{\theta}]$ where $\bar{\theta} > \underline{\theta} > 0$.[2] The types are distributed uniformly and are not observed by either firm. First, the two firms simultaneously select a quality, s_k $(k = A, B)$, from the interval $[\underline{s}, \bar{s}]$ where $\bar{s} > \underline{s} > 0$. To simplify the exposition, I will relabel the firms H and L where $s_H \geqslant s_L$. Since the firms are ex ante identical for every equilibrium where $A = H$ and $B = L$ there exists a mirror equilibrium where $A = L$ and $B = H$. Next, observing s_H and s_L, each firm simultaneously selects a price p_j. Finally, each consumer has the option to purchase one product. Thus, he can buy H's product, L's product, or neither. I assume that a firm produces to satisfy the demand for its good, given its declared price. A consumer of type θ_i receives a utility of

$$\theta_i s_j - p_j \tag{1}$$

from purchasing the good produced by firm $j(j = H, L)$ and zero if nothing is purchased.[3] An increase in s_j improves each consumer's utility. Hence, s_j can be thought of as the quality of j's product. Furthermore, one may interpret θ_i as consumer i's preference intensity for quality. A firm's profit is the revenue generated minus the total cost paid. Let the cost to j producing q_j units at a quality s_j be denoted $C(q_j, s_j)$. The standard vertical-product-differentiation model assumes that the cost is increasing in both quality and quantity, convex in quality, and the marginal cost of production is independent of quality. Thus, the standard model has $C(q_j, s_j) = cq_j + X(s_j)$ where X is a strictly increasing convex function; see McCannon

[2004] for an example. To introduce the trade-off between quality and quantity, assume instead that $C(q_j, s_j) = cq_j + aq_js_j$. The parameter $a \geqslant 0$ represents the magnitude of the trade-off. For larger values of a, a product with a greater quality is more costly, both in total cost and marginal cost. Also, the marginal cost of production is assumed to be constant for a given level of quality, $c + as_j$, and the marginal cost is greater when a higher quality product is made. I choose this specific functional form so that the impact of the quality–quantity trade-off can be readily observed. Furthermore, explicit solutions for price and quantity can be derived. Firm j earns a profit of

$$p_jq_j - C(q_j, s_j) \tag{2}$$

I focus on the set of subgame perfect Nash equilibria. As in Tirole [2002, p. 296], two assumptions are used to simplify the analysis.

Assumption 1 $\quad \overline{\theta} > max\left\{2\underline{\theta} - a, \frac{\underline{\theta} + a}{2}\right\}$

This assumption requires that there is enough heterogeneity relative to the cost parameter a so that the firms, when segmenting the market, are able to earn positive profits. This avoids the outcome of the high-quality firm driving price down towards marginal cost to capture the entire market. As will be shown, it also guarantees the existence of a pure-strategy equilibrium.

Assumption 2 $\quad \underline{\theta}\,\underline{s} \geq c + \frac{(\overline{s} - \underline{s})(\overline{\theta} - 2\underline{\theta})}{3} + \frac{a(\overline{s} + 2\underline{s})}{3}$

The second assumption ensures that firms choose prices so that every consumer prefers to make a purchase.

DERIVATION OF EQUILIBRIUM

Initially, consider the case where s_H is strictly greater than s_L, or, rather, consider the outcomes where the firms do not choose identical products. There is a threshold consumer type, denoted $\tilde{\theta}$, where the utility from purchasing H and L is equal. It follows from (1) that

$$\tilde{\theta} = \frac{p_H - p_L}{s_H - s_L} \tag{3}$$

Assuming that prices for the two goods are such that both firms receive positive sales (Assumption 1) and all consumers wish to make a purchase (Assumption 2), consumers with a preference intensity for quality greater than $\tilde{\theta}$ prefer to buy from H while consumers with a preference intensity for quality less than $\tilde{\theta}$ prefer to buy from L. As a result, the quantity sold by H is $\bar{\theta} - \tilde{\theta}$ while the quantity sold by L is $\tilde{\theta} - \underline{\theta}$. The profit functions, when determining the price, are

$$\begin{aligned} \pi_H &= \left(\overline{\theta} - \frac{p_H - p_L}{s_H - s_L}\right)[p_H - c - as_H] \\ \pi_L &= \left(\frac{p_H - p_L}{s_H - s_L} - \underline{\theta}\right)[p_L - c - as_L] \end{aligned} \tag{4}$$

Since I am considering subgame perfect Nash equilibrium, the firms choose prices by maximizing (4) taking s_H and s_L as fixed and requiring that consumers act optimally by using (3) as their decision rule. Therefore, in the pricing stage, the resulting prices are

$$(5)\qquad \begin{aligned} p_H^* &= c + (s_H - s_L)\frac{2\bar{\theta} - \underline{\theta}}{3} + \frac{a(2s_H + s_L)}{3} \\ p_L^* &= c + (s_H - s_L)\frac{\bar{\theta} - 2\underline{\theta}}{3} + \frac{a(s_H + 2s_L)}{3} \end{aligned}$$

An additional term is added to the standard result when $a > 0$ and quality and quantity are implicitly related through the cost function. Thus, fixing the quality levels, the stronger the impact quality has on the cost of production the greater the price. Furthermore, both prices increase in H's quality. An increase in s_H increases the differentiation of the products and allows both firms to charge a higher price. Similarly, the prices are decreasing in L's quality since an increase in s_L reduces the differentiation. At these prices, the threshold value of the preference intensity for quality is $\tilde{\theta} = (\bar{\theta} + \underline{\theta} + a)/3$. Therefore, the quantities the firms sell in equilibrium are

$$(6)\qquad q_H^* = \frac{2\bar{\theta} - \underline{\theta} - a}{3}, q_L^* = \frac{\bar{\theta} - 2\underline{\theta} + a}{3}$$

While a stronger relationship raises both firms' price (for fixed qualities) the trade-off moves sales from the high-quality producer to the low-quality producer. The firm producing the high-quality good has a relatively greater marginal cost of production and responds by reducing the quantity it supplies. Furthermore, the quantity each firm sells is independent of the quality chosen. Thus, prices fully adjust to the quality of the product so that the segmentation of the market remains the same. At these prices and quantities the firms' profits are

$$(7)\qquad \pi_H = (s_H - s_L)\left(\frac{2\bar{\theta} - \underline{\theta} - a}{3}\right)^2 \pi_L = (s_H - s_L)\left(\frac{\bar{\theta} - 2\underline{\theta} + a}{3}\right)^2$$

Consider the choice of quality by L first. An increase in s_L has two effects on L's profit. First, it reduces the differentiation of the goods, which increases the price competition driving down revenues. Second, an increase in s_L increases the cost. Both of these effects act to lower L's profit. Thus, $s_L^* = \underline{s}$. Firm H faces a trade-off — increasing quality raises its revenue but increases its cost as well. The increase in quality results in H selling to consumers with higher valuations for the good. Since the preferences for quality are assumed to be sufficiently varied in the population (Assumption 1), the price increase outweighs the cost increase. Hence, $s_H^* = \bar{s}$.

This derivation was performed assuming $s_H > s_L$. Suppose, instead, that $s_H = s_L$. A consumer prefers to buy from the firm with the lower price. Therefore, Bertrand price competition results in prices at marginal cost. Since $s_H^* = \bar{s}$ and $s_L^* = \underline{s}$ result in positive profits (Assumption 1), these are the equilibrium quality selections. Proposition 1 states the solution.

Proposition 1 In the subgame perfect Nash equilibrium outcome $s_H = \bar{s}$, $s_L = \underline{s}$, $p_H = c + (\bar{s} - \underline{s})(2\bar{\theta} - \underline{\theta})/3 + a(2\bar{s} + \underline{s})/3$, $p_L = c + (\bar{s} - \underline{s})(\bar{\theta} - 2\underline{\theta})/3 + a(\bar{s} + 2\underline{s})/3$, and consumers with $\theta_i < (\bar{\theta} + \underline{\theta} + a)/3$ buy from L while $\theta_i \geqslant (\bar{\theta} + \underline{\theta} + a)/3$ buy from H.

Proof. The subgame perfect Nash equilibrium can be determined by backwards induction. Consider, first, subgames where $s_H > s_L$. In the final stage, it is straightforward to see that given any p_H, p_L, s_H, and s_L ($s_H > s_L$), H is preferred to L by consumer i if $\theta_i \geqslant \tilde{\theta}$ and L is preferred to H by consumer i if $\theta_i < \tilde{\theta}$. By setting (1) greater than or equal to zero, each purchase is preferred to buying nothing if $\theta_j \geqslant p_j/s_j$. Therefore, in any subgame with $s_H > s_L$ consumer i prefers to buy from L if $\theta_i \in [max\{\underline{\theta}, p_L/s_L\}, \tilde{\theta})$, from H if $\theta_i \geqslant max\{\tilde{\theta}, p_H/s_H\}$, and from no firm otherwise.

Now consider the choice of p_H and p_L if $s_H > s_L$. If $q_L = \tilde{\theta} - \underline{\theta}$ and $q_H = \bar{\theta} - \tilde{\theta}$, then it follows immediately from (4) that p_H and p_L are as given in (5). Both prices are greater than the marginal cost if Assumption 1 holds. Also, at these prices every consumer buys a good if Assumption 2 holds. Furthermore, as a consequence of Assumption 2, $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$. Consider deviations. Since at these prices profits are maximized if $q_L = \tilde{\theta} - \underline{\theta}$ and $q_H = \bar{\theta} - \tilde{\theta}$, I need only consider deviations where either $q_L \neq \tilde{\theta} - \underline{\theta}$ or $q_H \neq \bar{\theta} - \tilde{\theta}$. Either firm could reduce its price and sell to all consumers. Of such prices the best deviation is to select the price where $\tilde{\theta} = \underline{\theta}$ for H or $\tilde{\theta} = \bar{\theta}$ for L (the highest price that captures the entire market), but then $q_L = \tilde{\theta} - \underline{\theta}$ and $q_H = \underline{\theta} - \tilde{\theta}$ (where one is zero) and profit is maximized by selecting the price that satisfies (5). Instead, L could increase its price so that not all consumers make a purchase; $p_L/s_L > \underline{\theta}$ where $q_L = \tilde{\theta} - p_L/s_L$. First, q_L decreases faster if $p_L/s_L > \underline{\theta}$ than if $p_L/s_L < \underline{\theta}$. Also, L's profit is concave when both $q_L = \tilde{\theta} - \underline{\theta}$ and $q_L = \tilde{\theta} - p_L/s_L$ and is decreasing in the former for values of p_L where $p_L/s_L < \underline{\theta}$. As a consequence, L's profit is less if selecting $p_L/s_L > \underline{\theta}$ than in (5). Furthermore, an increase in p_H by H either maintains $q_H = \bar{\theta} - \tilde{\theta}$ or, for high values of p_H, results in $q_H = 0$. Therefore, given Assumptions 1 and 2 the prices are as given in (5) if $s_H > s_L$.

Instead, suppose $s_H = s_L$. It follows from the standard identical-good, Bertrand game that $p_H = p_L = c + as_j$. At these prices, $\pi_j = 0$. Deviating to a greater price maintains a zero profit while deviating to a lower price generates a negative profit.

Now consider the choice of quality. As stated, if $s_H = s_L$ then $\pi_H = \pi_L = 0$. From (7) and Assumption 1, if $s_H > s_L$, then $\pi_H > 0$ and $\pi_L > 0$. Thus, neither H nor L selects $s_H = s_L$. Hence, it follows from (7) that, since L's profit is decreasing in s_L, $s_L^* = \underline{s}$ is the equilibrium choice. Also, H's profit is increasing in s_H so that $s_H^* = \bar{s}$. Therefore, in the subgame perfect Nash equilibrium outcome $p_H = c + (\bar{s} - \underline{s})(2\bar{\theta} - \underline{\theta})/3 + a(2\bar{s} + \underline{s})/3$, $p_L = c + (\bar{s} - \underline{s})(\bar{\theta} - 2\underline{\theta})/3 + a(\bar{s} + 2\underline{s})/3$, and, since $\tilde{\theta} = (\bar{\theta} + \underline{\theta} + a)/3$ and Assumption 2 holds, consumers with $\theta_i < \bar{\theta} + \underline{\theta} + a/3$ buy from L while $\theta_i \geqslant \bar{\theta} + \underline{\theta} + a/3$ buy from H. ■

Hence, a stronger quality–quantity trade-off results in fewer sales by the high-quality producer and more by the low-quality producer since it disproportionately affects the marginal cost of production of the high-quality producer. A stronger trade-off also pushes up the price of both goods. Again, the trade-off, captured by the a term, affects the marginal costs. An increase in the marginal cost of production results in an increase in the price of the good. Finally, as a consequence of Proposition 1, there are two subgame perfect Nash equilibria. In one firm A makes the choice of H and B selects as L. In a mirror equilibrium A makes the selection L and B chooses H.

The analysis assumes that $a > 0$ so that an increase in the quality of the product increases the marginal cost of production. Alternatively, one may think of examples where the opposite occurs. For example, experience at a task may improve quality so that the marginal cost of quality is less with more units produced. So long as the parameter a is not too small ($a > -(\bar{\theta} - 2\underline{\theta})$), the results continue to hold for negative values of a.

CONCLUSION

This note extends the standard vertical-product-differentiation model to incorporate a trade-off between quality and quantity inherent in certain goods. I show that the stronger this relationship, the more the sales shift from the high-quality producer to the low-quality producer since the former is disproportionately affected by the trade-off. This relationship has the impact of increasing both firms' prices.

Notes

1. See Peedin [2002] and Scott [2002] for discussions on the use of nitrogen to affect tobacco crops.
2. To simplify the analysis, I assume that $\bar{\theta}-\underline{\theta}=1$. Since this can be thought of as a renormalization of the variables it has no effect on the main results.
3. Assume that if indifferent between buying from H and L, the consumer selects H and, if indifferent between purchasing and not purchasing, a consumer opts to buy the good.

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