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Editor's Introduction

Introduction to complexity: emergence, graphs, and management studies

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The articles in this issue make use of the 'new science of complexity'. To some, complexity means networks; to others, it is a type of systems analysis that reveals sensitivity to initial conditions. To others, it is metaphorical or philosophical, with such terms as the 'edge of chaos', 'tipping points', 'long-tails', or 'butterfly effects' serving as catalysts for the analysis of very disparate phenomena.

The many uses of the term complexity can be criticized as suggesting a lack of definition. They also indicate a contrary claim. Complexity has already succeeded in penetrating the discourse and content of management and management research. Studies using complexity ideas and methods range from the qualitative and metaphorical to the quantitative and formal. Because of the wide applicability of complexity methods, the European Union has investigated heavily in research projects on complexity. These projects have focused, as Ralph Dum explains in this issue, on fundamental issues in the natural and social sciences, including innovation, diffusion, and epidemiology of disease. Often these projects consist of multi-disciplinary teams who struggle to cooperate and jointly learn. What do the many studies and projects share in common?

Complexity: more is different

Studies in complexity acknowledge the observation of Philip Anderson, the Nobel Prize winner in physics, that 'more is different'. This simple statement means that linear extrapolations are likely to fail. It also means that you can't count on the macro composition of social relations to be a mirror reflection of their micro foundations. On the other hand, it is not true that therefore the macro and the micro are decoupled. They are coupled, but the relationship is no longer symmetric.

The 'more is different' perspective is not philosophical as much as grounded in fundamental mathematic results that are found in many physical and social dynamics. For example, the polymath Herbert Simon explored the extreme value statistical distributions that arose from simple social behaviors (Simon, 1955). These insights found three decades later an unexpected application in the physics of internet growth and search that have been exploited by Google and many other search engines. Such important social phenomena as the Matthew Effect ('the rich get richer') appear in the pattern of internet links under the phrase of 'preferential attachment'. The emergence of social communities, such as open-source software and Facebook, are explicable by analyzing carefully how 'more' becomes 'different'. No matter the terminology, these important sociological ideas are firmly grounded in a body of mathematics.

The neglect of these mathematical results, which are well known to the earlier social network theorists of the 1950s and 1960s, has lead to a proliferation of empirical results that lack generality. We don't know whether what we find for a small network of monks holds for a large network of college students or whether one kind of network structure (i.e. topology) is better for diffusion than another kind of network. The social science of networks has become a list of empirical results, which are not consistent and are often answering the wrong questions. Complexity is a call to re-think our understanding of networks in reference to a set of new mathematical and conceptual tools rather than in reference to the standard linear expectation that a bit more of x and a lot more of x act proportionally on achieving some outcome.

Stuart Kauffman provides a useful example by which to understand how the science and mathematics of complexity and the 'more is different' concept can throw light onto important physical and social dynamics (Kauffman, 1995). Drop 30 buttons on to your table. Take a thread, choose two buttons randomly, and link them together. To make this exercise more relevant, we can let each button represent a neighbor in a small community and the threads indicate their friendships. How many times would you have to do this exercise until you can lift almost all the buttons by simply grabbing a single thread? Or how many friendships do you need to have until you can be pretty sure that a random assignment of friendships will generate a tightly knit community? In this example, more threads among the buttons lead suddenly to the network property of high connectivity. Thus, we find that social communities, such as 'You Tube' and 'Facebook', are illustrations of 'emergence'. We would like to answer, then, the Anderson question of when do more 'links' result in a qualitatively different graph or what management studies call a network.

We know the answer to this question due to the work of two Hungarian mathematicians named Paul Erdös and Alfred Rényi. The great discovery of Erdös and Rényi is that many important properties of random graphs appear suddenly. The particular property we are studying in the button exercise is the emergence of a 'giant component'. The transition from a fragmented network (or graph) to a connected graph happens very quickly; in physics, this transition is called a 'phase transition'. Erdös and Rényi showed that for a phase transition to occur, the probability, say of new links, must grow somewhat more quickly than a critical probability. Erdös and Rényi wrote many papers that launched the study of 'random graphs'.

The theories of random graphs are intriguing precisely because they have the quality that 'more is different'. Even by a random process of linking buttons together, there is a critical threshold that will be reached when a giant component is realized. This phase transition is the defining characteristic of what is called 'emergence'. Surely, emergence is a fundamental topic for social science and management. It would be a major achievement of social science to know when do demonstrations become revolutions, when do innovations become widely adopted, or how many links do you need to put into an organization in order to assure global coordination across activities and countries.

The phenomenon of emergence is an important example of the failure of linear thinking. Because networks evolve through non-linear interactions among parts (e.g. people in a group, components on a printed circuit board, agents in a supply chain), adding more interactions can result in a sudden and radical change in the property of the aggregate. In some cases, changing the density (i.e. the number of interactions among nodes or agents) can cause the evolution of social interactions to move towards a 'frozen' highly ordered network. This possibility of a society to become too ordered can be compared to a society which acts on the 'edge of chaos', that is, with enough order that people know each other or know how to find each other but not too much order that nothing is to be learned. A good example of such a network is a 'small world' that has a history of several decades of social science research and has exploded in importance since the publication of Watts and Strogatz's (1998) important work. You will also find a very engaging graphical presentation of how a change in density can move a network dynamically to a completely ordered graph at the website of Tom Snijder's Siena statistical program (http://stat.gamma.rug.nl/).

The digital treasure and the new science of networks

One major reason that graphs and complexity are becoming more important to social science and management studies is the confluence of ever-more powerful computers and the digitalization of content. The growing power of computers has long changed the methods and contents of research. The past decade has also seen the rapid increase in digitalized data. Digitalization is a radical complementary factor that permits the application of computational capacity to data never before available. We are living at the very start of this era in which text, music, and film are encoded digitally. We, as users, have a sharp understanding of the impact of these media on influencing powerfully culture and society. Oddly, in management studies, the potential of these technologies for research methods and opportunities has not yet been fully recognized.

It is remarkable that we sometimes fail to realize that digitalization is progressing every year, providing data on the size of cities a 1000 years ago, the trade relations among Roman towns, financial data on large and small corporations, and the integration of social and economic data on individuals and their social networks. The opportunities posed by the availability of these new and accumulating databases place new technical demands on the researcher.

The very large databases that are now available (e.g. patents, citations, financial data on firms, financial markets, scanner data in marketing, internet and email data) upset the usual rules of thumb. Social science has become accustomed to believe that a good statistical analysis leads us to accept or reject a hypothesis. Management articles have adopted this convention by an obsession with hypotheses and significance tests. However, when the database consists of 100,000 records, a significance test of 0.01 does not provide the information that we need to determine how well we have provided an answer. Nor does it make a lot of sense to be confident that a model that says $Y = 1.2 + 0.2X$ will forecast well in the context of a 'more is different'.

However, not all the databases that interest us are 'large'. Often, the data records that we have on social behavior and networks are big but not big enough that we can ignore possible problems of comparisons. One of the critical challenges in network analysis is simply to know whether a graph property (e.g. density, structural hole, path length) can be used to compare two graphs that differ dramatically in size. If both graphs are very large, we might be able to assume that asymptotically the network statistics converge to a common even if unknown distribution. The problem of comparison exists though for graphs (networks) that are small and medium size. What can we do then?

We cannot rely upon the standard significance tests because we don't know the distribution of the network statistics. However, the mathematics of Erdös and Rényi, among others, provides a useful benchmark in the identification of the appropriate random graph. For a few types of random graphs, we know the expectation of a statistic for a given number of nodes and links. For example, we can calculate the expected path length or clustering coefficient (which is closely related to the idea of a structural hole). More generally, we can estimate the statistics for any random graph through simulations that take the average of a statistic over many iterations. Once we know the expected statistic for a random graph of the same size, we can then compare it to the empirical estimation from the actual data. By this comparison, we can say that the empirical statistic is close to the random graph statistic, and hence not meaningful. Or if the empirical statistic is substantially far away, it can be argued that we are sufficiently confident that it is not random. Finally, we can also make the comparison of one empirical graph normalized by the random statistic to another graph which is normalized by its corresponding random graph. From this comparison, we can compare graph statistics across different networks of different sizes just like we can compare *Z*-score and other statistics across different samples of varying size.

It was this insight that Duncan Watts exploited brilliantly in his work on small worlds (Watts and Strogatz, 1998; Watts, 1999). Watts defined a small world as a case in which people are on average close to one another and yet they tend to be located in neighborhoods where everyone knows everyone. The mathematics of graphs calls the first short 'path lengths' and the latter 'clustered'. Watts initial interests were largely technical. He wanted to know how general are small worlds, in which general referred to the tendency for small worlds to exist on many types of topologies and network structures.

A graph topology defines the substrate or pattern of connections among nodes. For example, a ring lattice is the case in which people (nodes) are adjacent to two other nodes and the path is a closed chain, that is if you whisper a message to either neighbor, eventually the message comes back to you. A two-dimensional lattice is a grid in which every node is connected to, say, four other nodes and the overall pattern looks like a chessboard. When the edges are joined, the topology is said to be a torus.

Watts wanted to know if you start with a graph which has high clustering and high path lengths relative to the random graph, what will happen when the connections are randomly *rewired*. For many kinds of topologies, he performed this simulation of rewiring and observed the changes in the network statistics. He found that the path lengths quickly converge towards the lengths found for a random graph, but the clustering values remained high substantially longer. Figure 1 illustrates this behavior. On the vertical axis are the normalized values for path length and clustering. They are normalized by taking the empirical values of various kinds of topologies (e.g. a ring lattice) and dividing it by the expected values of these statistics (i.e. the path length and clustering coefficient) for a random graph.

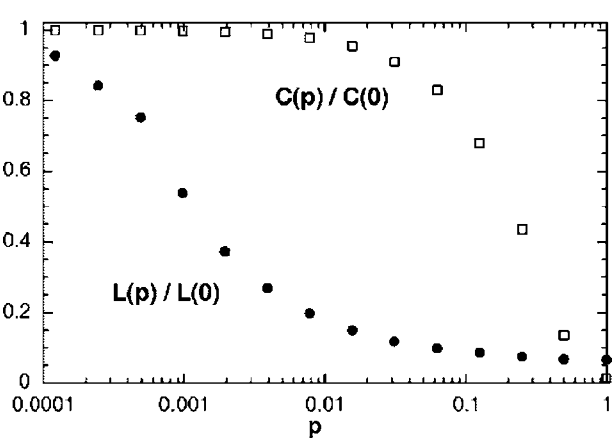

Figure 1 Behavior of network statistics in relation to the rewiring of links. The *Y*-axis is the normalized values for the empirical network statistics divided by the random values; the maximum value is arbitrarily set to 1. The *X*-axis is the log proportion of the links randomly reassigned in the network. $C(p)$ is the empirical clustering value that changes as a function of p (rewiring) and $C(0)$ is the fixed random value; $L(p)$ is the empirical path length as a function of p and $L(0)$ is the fixed random value. The graph shows that path length falls quickly to the empirical value, while the clustering value is more robust. The Small World domain is centered approximately on a P equal to 0.01.

The horizontal axis is the log of the proportion of links that are 'rewired', that is, a link from one node to another is assigned to another node. As rewiring increases, the randomization increases and we expected the empirical statistics to fall close to the random values. A small world characterizes the range in which the difference in the normalized clustering value and the normalized path length is greatest and substantial.

Watts developed this basic simulation to identify the important role played by short cuts in a network. What caused the path length to converge quickly to the random value was the creation of few edges that cut across the clusters. Ron Burt called these shortcuts 'structural holes' (Burt, 1992). In one perspective, Watts created a more general perspective by which to understand the relationship of clustering and path length, thus returning to an older tradition found in the work of Mark Granovetter (1973). By the use of simulations, Watts was able to show not only the generality of small worlds, but also to extend the concept to important topics in graph applications, such as games and diffusion.

The robustness of the findings across many types of topologies provides confidence that it is possible to compare clustering and path length statistics across networks, no matter the size of the networks or the difference in their topologies. For those engaged in empirical studies, Watts gave a methodology that allows the possibility to make more meaningful comparisons across networks. The method is quite simple because it is very familiar to the conventional statistical approach of standardizing estimates. For Watts, we estimate the empirical value and we standardize by the value of the random graph.

Gordon Walker and I exploited these ideas in the first paper published in a social science journal (besides Watts, 1999) to make use of these ideas (Kogut and Walker, 2001). We proposed the Small World statistic (SW) that is the result from dividing the normalized clustering coefficient by the normalized path length statistic. (Amaral, Tsochas-Reed, and Uzzi calls this the Q statistic in his article in his paper in this issue.) The SW statistic has been used in subsequent research, such as Baum *et al.* (2003), Davis *et al.* (2003), and Uzzi and Spiro (2005). Applying this idea to data on ownership links between German firms, we found that Germany is a small world. Subsequent work showed that many ownership networks for other countries are not small worlds (See the summary in Kogut and Walker, 2003a.) Thus it is not true that small worlds characterize all social networks. We then took the idea found in Watts' Figure 1 and applied it to the *empirical* data from the German network. These results are found in Figure 2. It is to be noted that the path length did not change, and the clustering coefficient begins to decay after many iterations. These results show then that the empirical network was close to the maximum small world, for the path length did not change and hence was close to the random value while the clustering value did change with rewiring. Of equal importance, the simulation showed that the network was *robust*, namely, it did not change very much despite substantial randomization. These simulation results indicating robustness coincided with similar findings of Albert *et al.* (2000) showing robustness for the internet despite rewiring. The finding of robustness counters claims made

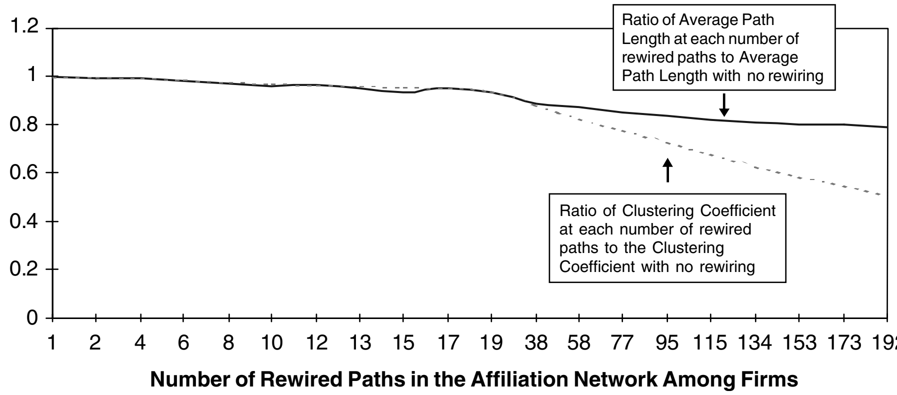

Figure 2 Results of simulating changes to German ownership patterns by 'Rewiring' paths in the affiliation network among firms. This graph implements the simulation strategy shown in Figure 1 to the German ownership data and provides an analysis of the effects of randomly rewiring the ownership ties among German firms. The rewiring is conducted thousands of times, and the path length and clustering values are averaged over the iterations. The simulations show that there is little change in the path length, implying that the empirical path length is already near the random expectation. The clustering value begins to deteriorate after 38 rewirings of the links, indicating that it was not initially at the value for a random graph. This difference in behavior supports the claim that the German network is a small world, where the empirical clustering value differs more from its random counterpart than the does the empirical path length from its random counterpart.

that the German network is rapidly changing and converging on the Anglo-Saxon model. (See the debate between Heinze, 2001, and Höpner and Jackson, 2002.) Our simulation results, bootstrapping from the empirical data, indicated that the German network would be very resistant to the inroads of globalization.

Watts and Strogatz's work spawned a very large literature. Since then, the basic ideas of been heavily criticized: the graphs are 'regular' (every node initially has the same number of links) even when empirically we see power law distributions; social networks are static despite that social networks evolve through the entry of new nodes and the formation of new links. (See Watts, 2004, for a review of these criticisms.) The work of Barabasi and his colleagues (see especially Barabasi and Albert, 1999) took a different though complementary approach from Watts and Strogatz in analyzing evolving networks that allowed for skewed degree distributions. They showed that a social rule of preferential attachment generates highly skewed distributions of links. Though this result has a longer history than they realized (such as Simon, 1955), it opened the door to a vast literature analyzing the dynamics of evolving networks.

One of the technical advances is the realization that many social networks are bipartite graphs (Newman *et al*., 2001.) A bipartite graph arises automatically from any affiliation network, by which it is meant that relationships among like-nodes are mediated by another set of nodes. For example, if Bobby and Jacques belong to the same jockey club, their relationship is mediated by their membership in this club. Similarly, German firms are not directly related (except in some cases) but they are related through common owners. Conversely, we can say German owners are related by investments in common firms. This observation might seem innocuous, but it is not. For obviously, a network in which firms have many owners (high degree) is different than one in which they have few owners (low degree). When we look at the *projection of the bipartite graph* (i.e. the adjacency matrix that represents the ties among firms), this difference in degree matters. After all, if firms have many owners, they will tend to be clustered, but whether this clustering is high or low compared to a random graph is unclear.

Newman, Strogatz, and Watts derived formulas for calculating several network statistics (e.g. path length, clustering, giant component) for bipartite graphs of arbitrary degree. Using the Davis, Yoo, and Baker data, they rejected that American boards of directors formed a small world once allowing for a bipartite topology. Kogut and Walker (2003b) applied Newman, Strogatz, and Watts bipartite formula for Poisson distributed degrees to their German data and found that the small world effect was still there but attenuated. Uzzi and Spiro (2005) also found that small worlds held for their data on broadway musicals. Conyon and Muldoon (2006) rejected the small world for German and UK corporate boards using the bipartite statistics.

In this issue, the very useful article by Uzzi, Amaral and Reed-Tsochas reviews the growing studies of small worlds in the management and social science literature. The most daring of the published results is the claim that performance of a node (e.g. a Broadway musical) is quadratic in the SW statistic. Other papers have not always been able to replicate the result. It might well be that such universal claims for the effects of a particular value of clustering may be sensitive to other statistics, such as the overall density of the graph. Statistical estimation of performance is not linear (as once claimed some articles) and it may or may not be quadratic. We will probably have to await more refined methods before answering these issues. The overall message of Uzzi *et al's* review is the revolution in understanding structural outcomes as the result of micro-rules, such as team assembly rules in the case of creating a Broadway musical. As Peter Hedström (2005) elegantly argues, determining mechanisms rather than causality is the methodological objective in complexity research in the social sciences.

Statistical analysis and simulations

The study of micro-rules and macro-structure speaks to the heart of social science research on analyzing levels of analysis. By asking what are the macro outcomes of local neighborhood structure, this line of inquiry permitted a reinvigoration of the statistical methods associated with Pip Pattison, Stanley Wasserman, and Tom Snijders among others. (For representative work, see Wasserman and Pattison, 1996; Snijders, 2002; Robins *et al.*, 2005; and Lazega *et al.*, 2006.) This statistical approach estimates the importance of types of local social relations (e.g. transitive, preferential attachment) in a network. Increasingly, these well-known tools in social science are diffusing in complexity studies.

Of course, the diffusion is often from the natural sciences to the social sciences. In the past decades, the tremendous effort in the exploration of the genome has lead to the development of many tools that permit new insight into social science investigation. In this issue, Balasz Vedres uses a particular tool, called sequence analysis, to study the pattern by which ownership structure evolved in Hungary. An important part of complexity studies is the understanding of patterns. For this reason, graphical representations (e.g. the familiar Pajek projections of networks that are frequently used) are very important tools in the understanding of complex relationships. Vedres' sequence analysis uncovers patterns in historical events, trying to reveal the pathways of evolution that are constrained by initial conditions.

The shadow of initial conditions on the evolution of a network has significant effects on estimation. Because of the difficulty of estimating in a 'more is different' context, statistical analysis is very sensitive to problems of convergence and initial conditions, as discussed earlier. Robins *et al.* (2005) analyzed explicitly the tendency of the statistics to be 'frozen' in certain domains of the joint distribution. As a result, simulations are essential part of statistical analysis in order to explore adequately the stability of the estimates and the limits to their applicability.

This combination of inference and simulation is fairly rare in social science research. It is however more familiar to us than we realize, for often the simulation is hidden from us and embedded in the statistical software that we use. A particularly interesting use of simulation can be conducted 'by hand' for very small samples. Thus the fascinating paper by Ragin and Sonnett (2005) applies a Boolean reduction to binary data from a small sample and then asks, what would be the causal inference if we were to add in cases that are not observed. They show that the causal results can change substantially, illustrating at the one time how sensitive inference is when causality is complex as well as showing how informed simulation can throw additional light onto causal explanations.

However, many simulations used in complexity are investigations without data and rely upon tools that would be hard to combine with statistical approaches. A good example of such simulation approaches are the so-called NK models used in two articles in this journal. The NK model uses a particular topology called the 'hyper-cube'. Figure 3 illustrates this cube for three dimensions; higher dimensions quickly become too difficult to represent graphically. The *N* refers to the number of dimensions, so that a dimension of 3 means an entity is defined by three properties. For example, a firm might be characterized as hierarchical or not, team organized or not, and piece rate incentivized or not. It should be clear that this representation is a Boolean graph, and thus the analysis shares some similarities with the Ragin and Sonnett approach (though this similarity has not been exploited). A Boolean graph of three dimensions will have 2^3 or eight unique combinations; each combination is a node in the graph. The NK model borrows the idea of fitness from biology and then associates a fitness value to each node. This projection simply means that a particular combination results in a specific performance (or fitness). Much like the analysis of Vedres, the components (or dimensions) will interact, and thus the fitness of any node will not be a linear sum of the individual components. The number of interactions is call *K*. Thus, the *NK* acronym stands for the number of dimensions (*N*) and the number of interactions among the dimensions or components (*K*).

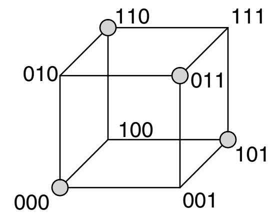

Figure 3 A hypercube in three dimensions. This figure represents a graph of a hypercube in which each node has three degrees and its position is denoted by three binary digits. The diameter of the graph is simply the maximum Hamming distance of 3 ($111-000=3$). The *NK* model works by assigning each node a fitness value, places agents across the nodes (though usually only a fraction of all possible nodes), and then allows the agents to search for better performance (indicated by the circles at some nodes) in reference to certain search rules and to the knowledge of other positions in the graph.

This interaction has an important effect on search for better fitness. For if a firm located at a node is not happy with its performance, it will move to another node in its *neighborhood* which will consist of three nodes. Each adjacent node will differ in just one component. If the effect of components on fitness were linear, a firm could find a path to best performance. However, the impact of nonlinearity caused by the interactions is to create the strong possibility of competency traps. The simulations, using an agent-based approach, allows agents to inhabit a node and then to rely on *local* knowledge to move to a better node when possible.

The strategy literature has borrowed the NK model from the sciences to ask what strategy or organizational design can a firm adopt in order to move to better performance. The dimensionality of any node forms a representation of the activities of a firm; the strategy is the search algorithm which the firm then uses. Two articles in this special issue investigate *NK* models in order to analyze organizational design and strategies to improve performance.

The *NK* model has proven to be an excellent metaphor for strategy, especially in the concept of a fitness landscape. This landscape, described in both articles, relates the activity (dimensions) of a firm to its environment (fitness). It thus stands in the classic strategy formulations of depicting strategic choice as understanding the match between the environment and the firm. The Caldart and Ricart article show how interdependencies among business units and strategic awareness (i.e. the completeness of local knowledge) influences performance. The article by Brusoni, Marengo, Prencipe, and Valente focus on the relationship between modularity and environmental uncertainty. Both of these articles exploit a new tool (NK simulation) to investigate the relation of strategy, structure, and the environment on performance.

As with any tool, there are weaknesses as well. One of the most important is that it is very difficult to match an NK model to data. Thus, most empirical investigations have relied on qualitative research that employed the terminology metaphorically, or on statistical data that often do not correspond to the stringent requirements of the model. The most stringent requirements are the imposition of myopic search (though clearly firms are able to access all kinds of information) and the technological determinism by viewing all possible fitness values as fixed by the dimensions. In this framing, the mechanics of the search is simply to allow firms to 'discover' better fitness than to 'invent' better solutions. Some of these objections can be met by considering competitive interactions, but then much of the clarity of results tends to decay quickly.

More broadly, an NK model relies upon the tools of graph theory: neighborhoods, path lengths, topology. Just as Watts asked whether small worlds are robust across many topologies, we might want to know if the NK results are robust for different topologies other than hypercubes. Equally so, we might want to know if the formal results from the mathematics of Boolean random graphs might serve as benchmarks to the results much like random graphs serve as a benchmark to the small world studies. The use of voting schemas has, for example, a long history in physical applications (such as in Ising models). It would be helpful if the formal results known in the physical sciences could be used to calibrate the simulation results in this burgeoning strategy literature.

The articles in this issue are a sampling of the work being conducted in management using ideas from complexity. This introduction offers the hope that continued exposure to concepts that have been found to be useful for the study of non-linear dynamics, evolving networks, and graphical representation will accelerate the pace of research in management studies. Many of the ideas used in the new science of complexity are drawn from a long tradition in social sciences that, at one time, was deeply impressed by the formal study of random graphs and empirical investigations. The magnificent studies of Anatol Rapoport (1957) that compared diffusion in social networks to random graphs, or the profound reflections of Herbert Simon (1962) on complexity gradually faded into the background. In the past decade, this tradition has been given new life as well as new tools that are now diffusing across the natural and social sciences. (See for example, the discussion of Patrick Cohendet and Patrick Llerena in the last issue.) One should be skeptical that this diffusion promises an enduring era of multi-disciplinarity. However, for the time being at least, complexity research has united many disciplines in the study of social phenomena. This issue represents a small sample of the importance of this cross-disciplinary discussion for research in management.

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