Papers

Asset allocation versus security selection: Evidence from global markets

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Abstract One of the most debated issues of investment management is the relative importance of asset allocation and security selection, and the overwhelming consensus is that asset allocation is more important. This paper argues that many investors have a false impression of the relative importance of these activities, because they fail to distinguish between the consequences of investor behaviour and the opportunity set offered by the capital markets. A methodology is applied that controls for investor behaviour and isolates the respective opportunity sets associated with asset allocation and security selection. Contrary to the widely held view, it turns out that choosing stocks within the equity component of a portfolio is substantially more important than choosing a portfolio's exposure among stocks, bonds and cash. Further evidence is provided of the dominant importance of security selection by applying option pricing theory to value asset allocation skill and security selection skill. The results, taken together with earlier studies of the historical impact of security selection on fund performance, provide compelling evidence that investment managers compress the natural distribution of opportunities available from security selection.

Keywords: *asset mix policy; timing; security selection; investor behaviour; wealth dispersion; utility dispersion; exchange option; relative volatility; Monte Carlo simulation; log-wealth utility; natural opportunity set*

Introduction

One of the most debated issues of investment management is the relative importance of asset allocation and security selection, and the overwhelming consensus is that asset allocation is more important. This paper argues that many investors have a false impression of the relative importance of these activities, because they fail to distinguish between the consequences of investor behaviour and the opportunity set offered by the capital markets. A methodology is applied that controls for investor behaviour and isolates the respective opportunity sets associated with asset allocation and security selection. Contrary to the widely held view, it turns out that choosing stocks within the equity component of a portfolio is substantially more important than choosing a portfolio's exposure among stocks, bonds and cash.

The paper is organised as follows. First, the rationale behind the received doctrine that asset allocation is more important than security selection is discussed, and the relevant literature is reviewed. Then a simple mathematical model of relative importance is presented based on a limited set of asset classes and securities. Next, historical asset class and security returns are bootstrapped from five countries to determine the dispersion in wealth arising from variation in asset class exposure and the dispersion arising from variation in security exposure. Option pricing theory is then applied to quantify the value of asset allocation skill and security selection skill. The paper concludes with several potential objections to the analysis along with responses and a summary.

The received doctrine

The widely held view that asset allocation is more important than security selection arises in part from a study by Brinson *et al*. (1986) called 'Determinants of Portfolio Performance'. In that study, the authors attributed the performance of 91 large corporate pension plans to three investment activities: policy, timing and security selection. They defined the policy return as the return of the long-term asset mix invested in passive asset class benchmarks. They then measured the return associated with deviations from the policy mix assuming investment in passive benchmarks and attributed this component of return to timing. Finally, they measured the return associated with deviations from the passive benchmarks within each asset class and attributed this component of return to security selection. For each of the 91 funds, they regressed total return through time on these respective components of return. These regression analyses revealed that asset allocation policy on average across the 91 funds accounted for 93.6 per cent of total return variation through time and in no case less than 75.5 per cent. Brinson *et al.* (1991) updated this study and found that asset allocation policy still accounted for more than 90 per cent of return variation.

Ibbotson and Kaplan (2000) published an article that distinguished cross-sectional return variation from intertemporal return variation. They confirmed the Brinson *et al.* result that asset mix policy accounts for more than 90 per cent of return variation through time, but they demonstrated that only 40 per cent of return differences across funds is attributable to asset mix policy.

Hensel *et al.* (1991) acknowledged the importance of investor behaviour, but they did not control for this effect. They showed that asset allocation could appear more or less important depending on the reference point around which it varies.

Ankrim and Hensel (2000) objected to the Brinson *et al.* methodology, because it attributes the returns from 0 per cent up to the policy portfolio return to asset allocation. This attribution assumes implicitly that the default exposure is 100 per cent cash.

Jahnke (2000) argued that it is difficult to measure the importance of asset allocation because the answer depends on many factors such as the extent to which investors engage in active asset allocation and security selection, investment expenses and skill. This paper also raised the important issue that investor behaviour may confound the implications of historical results, but it made no attempt to solve this problem.

Finally, Statman (2000) endorsed the notion that asset mix policy is critical, because he argues investors are not skilful at market timing or security selection.

The present authors find nothing inherently wrong with the Brinson *et al.* studies. The studies provide correct answers to the questions posed by their authors, which dealt with decomposition of historical performance. Nor do they disagree with the observations cited above, which raise valid points regarding potential misinterpretation of the Brinson *et al.* results. The present authors' view is that, in order to assess the relative importance of asset allocation and security selection, it is necessary to move beyond the historical performance of actual funds, because these results depend on two separate influences: the investment opportunities available from variation in asset class and security returns, and the extent to which investors chose to exercise discretion in exploiting these opportunities. The Brinson *et al.* studies and others like them present a joint test of investor behaviour and capital market opportunities. They do not answer the question 'Which activity is more important: asset allocation or security selection?' This is what this paper proposes to do. But first the question will be framed in mathematical terms.

A mathematical model of relative importance

What is meant by importance? Importance is defined as the extent to which a particular investment activity causes dispersion in wealth. Dispersion is important to investors who believe they possess or can acquire skill, because it enables them to increase wealth beyond what they could expect to achieve by passive investment or from average performance. Dispersion is also important to investors who are unskilful because it exposes them to losses that might arise as a consequence of bad luck. As beneficial as it is for skilful investors to focus on activities that cause dispersion, it is equally important for unskilful investors to avoid activities that cause dispersion.

In this simple mathematical model, the potential for dispersion is measured as the relative volatility between two investments[1]; in this example, investments that differ by security composition or investments that differ by asset class composition. Should one have any priors about the effect of asset allocation and security selection on dispersion? One might expect security selection to cause greater dispersion than asset allocation because individual securities are more volatile than the asset classes that comprise them unless the securities move in perfect unison. Therefore, if it is argued that asset allocation causes greater dispersion, one necessarily believes that high correlations among individual securities offset their high individual volatilities.

Consider two asset classes that contain two securities each. Asset class A includes securities A1 and A2, while asset class B includes B1 and B2. The relative volatility and hence the importance of security selection within asset class A is measured as shown:

$$\sigma\varepsilon_{A1,A2} = (\sigma_{A1}^2 + \sigma_{A2}^2 - 2\rho\sigma_{A1}\sigma_{A2})^{1/2} \qquad (1)$$

where $\sigma\varepsilon_{A1,A2}$ is the relative volatility between A1 and A2, σ_{A1} is the standard deviation of A1, σ_{A2} is the standard deviation of A2, and ρ is the correlation between A1 and A2.

Table 1 Standard deviation, correlation and relative volatility

	Standard deviation (%)	Correlation (%)	Relative volatility (%)		Standard deviation (%)	Correlation (%)	Relative volatility (%)
A1	10.00			A1	10.00		
A2	10.00	0.00	14.14	A2	10.00	50.00	10.00
B1	10.00			B1	10.00		
B2	10.00	0.00	14.14	B2	10.00	50.00	10.00
A	7.07			A	8.66		
B	7.07	0.00	10.00	B	8.66	50.00	8.66
A1	10.00			A1	10.00		
A2	10.00	50.00	10.00	A2	10.00	50.00	10.00
B1	10.00			B1	10.00		
B2	10.00	50.00	10.00	B2	10.00	50.00	10.00
A	8.66			A	8.66		
B	8.66	33.33	10.00	B	8.66	25.00	10.61

The same equation is used to calculate the relative volatility between securities B1 and B2.

The importance of choosing between asset class A and asset class B is measured in the same way, but first the standard deviation of each asset class must be calculated. If one assumes the individual securities are weighted equally within each asset class, the standard deviation of asset class A equals:

$$\sigma_A = (\sigma_{A1}^2 \times 0.5^2 + \sigma_{A2}^2 \times 0.5^2 + 2\rho\sigma_{A1} \times 0.5 \times \sigma_{A2} \times 0.5)^{1/2} \quad (2)$$

where σ_A is the standard deviation of asset class A, σ_{A1} is the standard deviation of A1, σ_{A2} is the standard deviation of A2, and ρ is the correlation between A1 and A2.

We repeat the same calculation to derive the standard deviation of asset class B.

The relative volatility between asset class A and asset class B equals:

$$\sigma\varepsilon_{A,B} = (\sigma_A^2 + \sigma_B^2 - 2\rho\sigma_A\sigma_B)^{1/2} \quad (3)$$

where $\sigma\varepsilon_{A,B}$ is the relative volatility between A and B, σ_A is the standard deviation of A, σ_B is the standard deviation of B, and ρ is the correlation between A and B.

Suppose the four securities are uncorrelated with each other. Then security selection would be more important than asset allocation because the asset classes would be less risky than the average risk of the securities they comprise, which results in less relative volatility between the asset classes than between the securities within each asset class. Moreover, as more securities are added, the asset class standard deviations decline further, which in turn further reduces the relative volatility between the asset classes. If, for example, security returns are uncorrelated and the securities are equally weighted, then the asset class standard deviation diminishes with the square root of the number of securities included. It is only when the correlation between asset class A and B is substantially less than the correlation between the individual securities within the asset classes that the relative volatility between asset classes is greater than the relative volatility between securities. These relationships are illustrated in Table 1.

The upper left panel shows that relative volatility between asset classes is less than the relative volatility between securities when they are all uncorrelated with one another. The upper right panel shows the same result when they all are equally correlated with one another. The lower left panel shows the asset class and security correlations that lead to convergence between relative volatilities. Finally, the lower right panel provides an example in which the relative volatility between asset classes is higher than it is between securities.

The associations between standard deviation, correlation and relative volatility are easy to illustrate when only two asset classes are considered, each divided equally between only two securities. These associations become less clear when several asset classes are considered, weighted differently among hundreds of securities with a wide range of volatilities and correlations. Under these real-world conditions, it is easier to resolve the question of relative importance by a simulation procedure known as bootstrapping.

Resolution by bootstrapping

Bootstrapping is a procedure by which new samples are generated from an original dataset by randomly selecting observations from the original dataset. It differs from Monte Carlo simulation in that it draws randomly from an empirical sample, whereas Monte Carlo simulation draws randomly from a theoretical distribution.

This dataset includes the individual stock returns of the MSCI equity indexes in Australia, Germany, Japan, the UK and the USA, as they were constituted at the end of 2001, as well as the returns of the JP Morgan government bond indexes and the JP Morgan cash indexes in these countries.[2] It does not include individual security returns for bonds or for cash instruments. Calendar year returns are used from January 1988 to December 2001.

The security selection bootstrap proceeds as follows for each country.

1 Randomly select a stock from the MSCI sample and calculate its total return.
2 Replace the randomly selected stock into the original sample.
3 Again, randomly select a stock from the MSCI sample, calculate its total return, and replace it.
4 Continue to select stocks randomly with replacement until 100 stocks are chosen in order to obtain a diversified stock portfolio.
5 Calculate the average total return of the 100 selected stocks.
6 Compute a portfolio return comprising a 60 per cent allocation to the randomly selected stocks, a 30 per cent allocation to the bond index, and a 10 per cent allocation to the cash index.
7 Each year repeat steps 1–6 1,000 times.
8 Calculate the annualised cumulative returns of the 1,000 portfolios and then rank them.

This bootstrapping procedure produces cumulative returns over 14 years for 1,000 portfolios whose stock allocation is fixed at 60 per cent but whose individual stocks are selected randomly each year. The replacement rule allows stocks to be selected more than once; thus the individual stock weights can range from 1 per cent to 100 per cent within the fixed 60 per cent stock allocation.

The asset allocation bootstrap proceeds similarly for each country.

1 Randomly select the equally weighted MSCI stock index, the JP Morgan government bond index, or the JP

Morgan cash index from a sample that is weighted 60 per cent toward the stock index, 30 per cent toward the bond index, and 10 per cent toward the cash index. Then calculate its total return.
2 Replace the randomly selected asset into the original sample.
3 Again, randomly select the equally weighted MSCI stock index, the JP Morgan government bond index or the JP Morgan cash index from the weighted sample, calculate its total return, and replace it.
4 Continue to select assets randomly with replacement until 100 assets are chosen.
5 Calculate the average total return of the 100 selected assets.
6 Each year repeat steps 1–5 1,000 times.
7 Calculate the annualised cumulative returns of the 1,000 portfolios and then rank them.

This bootstrapping procedure produces cumulative returns over 14 years for 1,000 randomly selected asset portfolios in which the component securities are fixed. Therefore, the return variation among the 1,000 portfolios arises purely from random variation of the asset mix each year.

What could be simpler? The importance of security selection is measured by holding a constant asset mix at a 60/30/10 allocation among stocks, bonds and cash, and calculating variation in return due purely to variation among randomly diversified stock portfolios. Then the importance of asset allocation is measured by holding constant individual security weights and calculating variation in return owing purely to asset allocation variation around an expected allocation of 60/30/10 to stocks, bonds and cash respectively. And the answer is …?

The answer

Random variation among individual securities within the stock component of a portfolio causes substantially more return variation than does random asset allocation among stocks, bonds and cash, holding constant the individual security weights within the stock component. Figure 1 shows the extent to which a talented investor (top 25th or 5th percentile) would improve upon average performance by engaging in asset allocation and security selection. It also shows how much below the average performance an unlucky investor (bottom 75th or 95th percentile) would perform depending on the choice of investment discretion.

The dispersion around average performance arising from security selection is substantially greater than the dispersion arising from asset allocation in every country, and it is particularly large in the US because the US has a larger number of individual securities.

One might argue that annualised cumulative return is not an appropriate metric because it ignores risk. This concern is addressed in Figure 2. Rather than rank the 1,000 asset allocation and security selection portfolios by annualised return, they are ranked by utility, which encompasses both return and risk. Specifically, a mean–variance approximation of log-wealth utility is used, as shown.[3]

$$U = \ln(1 + \mu) - (1/2\sigma^2)/[(1 + \mu)^2] \quad (4)$$

where U is the utility, ln is the natural logarithm, μ is the annualised return, and σ is the annualised standard deviation.

Figure 2 shows the dispersion of utility associated with random asset allocation and security selection. In this case, the asset allocation and security selection portfolios are ranked by utility to determine their percentile rankings. In

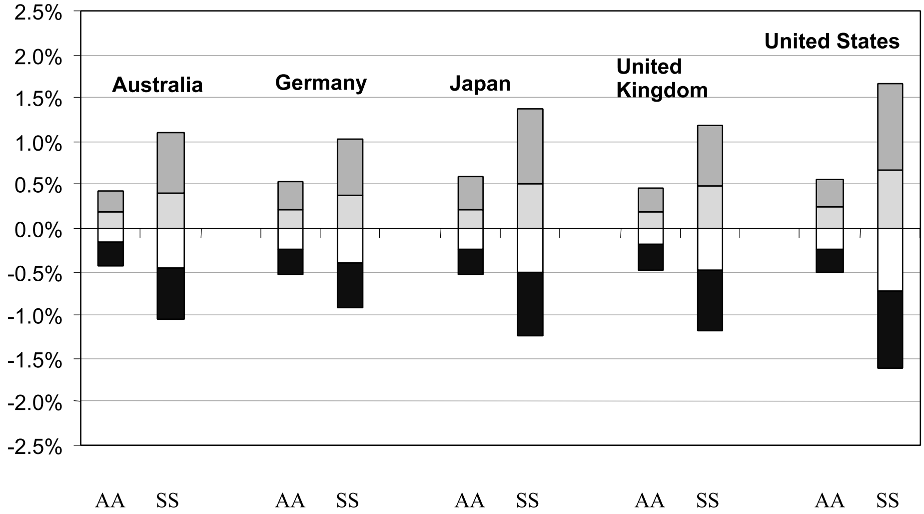

Figure 1 Asset allocation versus security selection 5th, 25th, 75th and 95th percentile performance annualised difference from average (1988–2001)

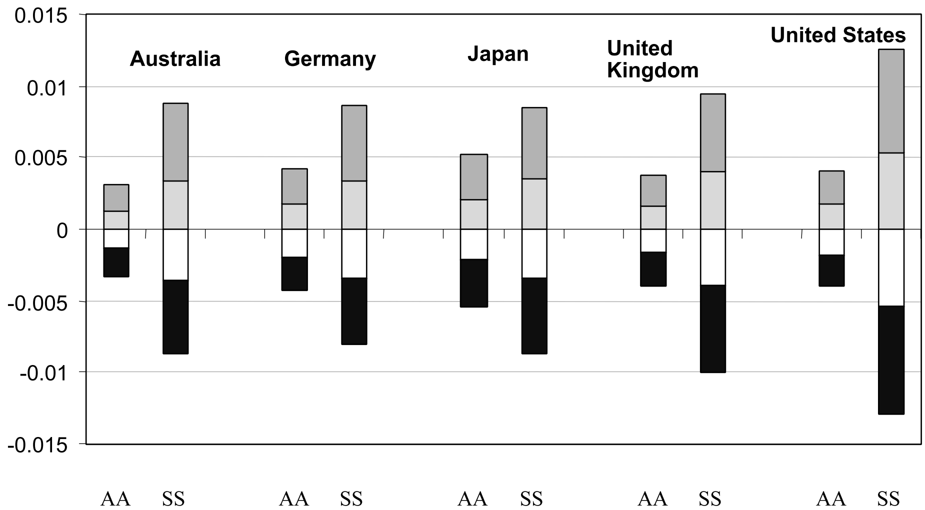

Figure 2 Asset allocation versus security selection 5th, 25th, 75th and 95th percentile utility annualised difference from average (1988–2001)

either case, the dominant importance of security selection is confirmed. The dispersion of utility, as well as return, is much greater for those portfolios that vary by their security composition than it is for those portfolios that vary by their asset class composition. Table 2 shows the specific values associated with Figures 1 and 2.

These simulations offer compelling evidence of the dominant importance of security selection. The next section assigns specific values to asset allocation skill and security selection skill.

The value of asset allocation and security selection skill

The value of asset allocation skill and security selection skill is measured by employing a variation of the Black–Scholes option pricing model to

Table 2 Specific values associated with Figures 1 and 2

Percentiles	Australia AA	Australia SS	Germany AA	Germany SS	Japan AA	Japan SS	UK AA	UK SS	US AA	US SS
	Annualised return 1988–2001. Difference from average (%)									
5	0.42	1.10	0.53	1.02	0.58	1.37	0.46	1.18	0.55	1.66
25	0.18	0.41	0.22	0.39	0.22	0.51	0.19	0.50	0.24	0.67
75	−0.16	−0.46	−0.24	−0.40	−0.23	−0.50	−0.19	−0.49	−0.24	−0.73
95	−0.42	−1.06	−0.53	−0.93	−0.55	−1.24	−0.47	−1.18	−0.52	−0.61
	Annualised utility 1988–2001. Difference from average (%)									
5	0.0031	0.0088	0.0043	0.0087	0.0052	0.0084	0.0038	0.0095	0.0041	0.0126
25	0.0014	0.0034	0.0018	0.0035	0.0021	0.0036	0.0016	0.0040	0.0019	0.0054
75	−0.0013	−0.0036	−0.0019	−0.0034	−0.0021	−0.0034	−0.0016	−0.0040	−0.0018	−0.0053
95	−0.0032	−0.0086	−0.0043	−0.0080	−0.0053	−0.0086	−0.0039	−0.0099	−0.0039	−0.0128

value an exchange option.[4] The owner of an exchange option has the right to exchange one risky asset for another. This case wishes to determine the value of an option to exchange median performance for top quartile performance and the value of an option to exchange bottom quartile performance for median performance.

The value of an exchange option is given by Equation (5), assuming income is reinvested and the starting portfolio values are 1.00.

$$EO = N(d_1) - N(d_2) \quad (5)$$

where EO is the value of exchange option, $d_1 = (\ln(V_P/V_M) + 1/2\sigma\varepsilon^2 t)/(\sigma\varepsilon\sqrt{t})$, V_P is the starting value of chosen percentile portfolio, V_M is the starting value of the median portfolio, $N(.)$ is the cumulative normal probability, ln is the natural logarithm, $\sigma\varepsilon$ is the relative volatility between V_P and V_M, t is the time remaining to expiration as a fraction of a year, $d_2 = d_1 - \sigma\varepsilon\sqrt{t}$.

Table 3 shows the option premiums associated with asset allocation and security selection skill, based on the relative volatility of the simulated performance. For example, the value of an option to exchange median asset allocation performance for top quartile asset allocation performance within the US markets equals 0.63 per cent of the portfolio's asset value. Top quartile skill as a security selector within the US markets by comparison is worth 2.70 per cent, more than four times as much. In all countries, the option to acquire top quartile performance from security selection is more valuable than the option to acquire top quartile performance from asset allocation. It is hard to argue that asset allocation is more important if security selection skill is more valuable.

Table 3 also reveals the value of an option to exchange bottom quartile performance for median performance. Although one could simply avoid bottom quartile performance by avoiding risk in a particular activity, the value of such an exchange option is a good indicator of how important it is to avoid activities that expose investors to bad luck. Again, the option premium placed on the avoidance of bad luck is significantly more valuable for security selection than for asset allocation in all countries. It ranges from 2.11 times as valuable in Germany to 3.69 times as valuable in Japan.

Objections

How might one object to this analysis?

— Only security selection is allowed within the stock component.

Table 3 Value of an option to exchange median performance for top quartile performance and bottom quartile performance for median performance

	Australia	Germany	Japan	UK	US
			Asset allocation (%)		
Top quartile	0.79	0.78	0.76	0.61	0.63
Bottom quartile	0.43	0.64	0.45	0.50	0.81
			Security selection (%)		
Top quartile	1.91	0.93	1.53	2.33	2.70
Bottom quartile	1.14	1.35	1.66	1.63	2.01
			Relative value		
Top quartile	2.42	1.19	2.01	3.82	4.29
Bottom quartile	2.65	2.11	3.69	3.26	2.48

True, but greater discretion to vary security exposure within the bond and cash components would only amplify the dominant influence of security selection.

— The stock universes are limited to the MSCI indexes, and in early years attrition reduces these universes even further.

True, but a larger universe would enhance the opportunity to affect wealth by security selection, because the volatility of asset classes declines as more securities are included.

— Because only stocks that have survived from 1988 to 2001 are included, the stock sample is biased.

The stock sample may or may not be biased. Moreover, to the extent the performance of the deleted stocks has been greater or worse than the surviving stocks, the simulation understates the potential impact of security selection on wealth.

— The choice of equally weighted indexes biases the results against asset allocation.

Capitalisation weighted indexes are less volatile than equally weighted indexes. It is therefore probably true that capitalisation weighting would reduce the relative volatility among asset classes, but it would also reduce the relative volatility among portfolios that differ by security composition. It is not clear which effect would dominate, but it is unlikely that the net effect would change the results significantly.

— The simulated stock portfolios represent unduly risky portfolios that few investors would be willing to own.

False. The randomly selected security portfolios are well diversified. For example, the average tracking error of the top and bottom quartile portfolios that vary by security weights relative to the median portfolios equals 4.31 per cent. It ranges from a low of 2.32 per cent in Germany to a high of 6.78 per cent in the US. Moreover, the same pattern prevails when the dispersion of utility is measured, which incorporates risk. If the portfolios that vary by security exposure were so risky, one would expect to find a substantial compression in the distribution of utility, which one does not find.

— The portfolios that vary by security composition contrast the best stocks against the worst stocks and therefore represent unrealistic extremes.

False. The securities are not first ranked from best to worst and then the top and bottom portfolios identified. The top 5th percentile and 25th percentile are contrasted with the bottom 95th and 75th percentiles based on random selection. Hence these portfolios reflect the dispersion in performance that would occur naturally if investors did not consciously restrict tracking error.

— There may be greater opportunity to add value through asset allocation than through security selection because there is less arbitrage among asset classes than among individual securities.

It may be true that asset classes are priced less efficiently than securities and investors are more likely to succeed at asset allocation than by choosing among individual securities. If so, investors should indeed focus their resources on asset allocation, not because it is more important but because it offers a higher probability of success. The greater dispersion that arises from security selection combined with lack of skill simply points to the importance of avoiding security selection.

Conclusion

Many investors have concluded falsely that asset allocation is more important than security selection because performance attribution studies confound investment opportunities with investor behaviour. It is argued that the dominance of asset allocation in explaining past performance only reflects the industry's unwillingness to engage meaningfully in security selection. Moreover, this unwillingness is interpreted as persuasive evidence of the dominant importance of security selection. Because security selection is so important, unskilful but nonetheless perceptive investors choose to avoid it. Moreover, lest there be any doubt about relative importance, exchange options are significantly more valuable when applied to portfolios with variable security weights than to portfolios with variable asset class weights.

Finally, the simulation results, taken together with actual fund performance, highlight a potentially unrecognised feature of the institutional investment industry. Security selection strategies do not offer as wide a distribution of opportunities as the distribution that arises naturally from security returns. The authors suspect that managers compress the natural opportunity set to reduce their exposure to business risk.

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Notes

1 Relative volatility in many applications is called tracking error.
2 The authors do not have individual stock data for these indexes as they were constituted each year. Therefore, the sample suffers from attrition as returns are calculated for earlier years.
3 A log wealth utility function assumes utility is equal to the logarithm of wealth, which implies that utility increases with wealth but at decreasing rate. It is one of a family of utility functions that assume investors have constant relative risk aversion.
4 Exchange options were the first exotic options. The first model to value exchange options was proposed by Margrabe (1978).

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