Journal of the Operational Research Society (2009) 60, S140–S148

www.palgrave-journals.com/jors/

Locational analysis: highlights of growth to maturity

HK Smith[1]*, G Laporte[2] and PR Harper[3]

[1]University of Southampton, Southampton, UK; [2]HEC Montréal, Montreal, Canada; and [3]Cardiff University, Cardiff, UK

Locational analysis has grown to maturity over the last decades, from its earliest roots, to fruitfulness in a wide-ranging number of strands that join with other disciplines and applications such as environmental planning and supply chain management. This paper charts the progress of location theory in three stages: a period of early contributions, when a number of seminal geometrical and geographical problems were studied; a 'coming of age' with the development of defining or classical problems that have proved fundamental to much later research and a third period of new models and new applications.

Journal of the Operational Research Society (2009) **60,** S140–S148. doi:10.1057/jors.2008.172
Published online 4 February 2009

Keywords: location; history of OR

Introduction

Locational analysis is a specialized branch of combinatorial optimization that has grown from early foundations to maturity, with most growth occurring since the 1960s. A wide range of problems has emerged, which may be characterized in general as finding optimal locations for facilities, both for business purposes and in public service. Candidate facility sites may be specified as discrete sites of choice, along networks or continuously in a plane. A comprehensive survey of location problems is given by ReVelle and Eiselt (2005). Taxonomies of the location field are provided by ReVelle *et al* (2008), with reviews of recent discrete models. Daskin (2008) illustrates different types of location modelling with examples in the field of discrete problems.

Location theory is an interdisciplinary field rooted in mathematics, computer science, operational research, economics, geography and other areas. We aim in this paper to describe something of the rich diversity of theoretical research and applications born out of the varied nature of this subject base.

We present a number of highlights of the development of location problems for the interested non-specialist reader. We do not attempt to present anything approaching a complete survey of the field, nor an exact taxonomy of the problems that have been tackled, but to give a concise but representative selection of what has emerged. As milestone publications in this field, we refer to early works in different areas and significant developments. In addition, we point to review papers and applications of interest.

We start fairly briskly with the beginnings of the subject, giving roots of some regions of study but before an identifiable body of locational research can be discerned. In the second section, we continue with descriptions of a number of location problems that represent the 'coming of age' of the subject. During this period, classical location models were stated and cast in the framework of discrete optimization. These models were often of a simplistic nature, but they are those from which a great deal of the following research has been derived. The third epoch starts in the early 1980s and corresponds with the advent of a series of international symposia in locational analysis. This is when the field truly started to mature with the emergence of new applications and the development of sophisticated models and algorithms.

Period I: Early contributions

Early roots of locational analysis are provided by several works of a geometrical or geographical nature: the way is pointed to modern locational research in the continuous plane. We look briefly at the 17th century, move on to consider Alfred Weber's work in the early 20th century, and follow on with Voronoi and Delauney's divisions of the plane. We finish with the root of competitive location brought about by Hotelling.

It may be argued that location analysis originated in the 17th century with Pierre de Fermat's (1601–1665) problem: 'given three points in the plane, find a fourth point such that the sum of its distances to the three given points is a minimum' (Kuhn, 1967). Evangelista Torricelli (1608–1647) is one of those credited with the geometrical construction needed to find such a spatial median or 'Torricelli point'; details are given by Drezner *et al* (2002). However in the last century, the 'Weber Problem' of Alfred Weber (1909) begins the era of modern location analysis, with its application to industrial location and many subsequent extensions (Drezner *et al*, 2002). The Weber problem finds the point in a plane that

*Correspondence: HK Smith, School of Mathematics, University of Southampton, Southampton S017 1BJ, UK.
E-mail: Honora.Smith@soton.ac.uk

minimizes the sum of weighted Euclidean distances to a set of fixed points. This is interpreted as finding the factory location that minimizes the total weighted distances from suppliers and customers, where weights represent relative volumes of interactions, for example, weight of material to be transported from a supplier, or volume of finished products for a customer.

Voronoi diagrams or tessellations (Voronoi, 1907) divide up a planar region into small subregions, according to closeness to a discrete set of points. Such decompositions of a metric space have been used in heuristics for subsequent continuous location problems (Suzuki and Okabe, 1995). Delaunay's triangulation method, for set of points in a plane, gives a net of triangular regions such that no point is contained in any triangle's circumcircle (Delaunay, 1934). This method has proved useful in the creation of efficient algorithms. The Weiszfeld (1937) algorithm, a least squares method with iteratively changing weights, converges on optimal solutions for many problem instances in continuous location. This algorithm has provided a basis for subsequent research into heuristics.

We complete our epoch of the roots of location theory with a mention of the work of Hotelling (1929), who considers relative equilibrium positions of competing businesses, on a straight line and also in the plane, with price variations. Although based on greatly simplifying assumptions of costs and demand, this work lays the foundations of future competitive location modelling, to emerge later in our timeline.

Period II: Coming of age

It is only in the 1960s and 1970s, with wide availability of computing power for processing and analysing large amounts of data, that we see the real beginnings of modern optimization, followed by the accompanying research into location problems. This we propose as the maturing or 'coming of age' period of locational analysis, mainly devoted to the study of the classical p-median, p-centre, location-covering, simple plant location and quadratic assignment problems (QAPs) and their extensions. It is also during this period that the discipline of regional science takes shape, blending together location theory, economics and regional development; one of its leading proponents is Isard (1969). The twin survey by Tansel *et al* (1983a, b) covers this period.

At this time, it is Cooper (1963, 1964) who extends Weber's single-facility problem to begin the multi-facility location–allocation problem. Maranzana (1964) then moves the problem from continuous space to networks, with what has become a classical local search heuristic. However, it is Hakimi (1964, 1965) who completes the foundation of research into the p-median and other problems on a network, made possible by the efficient algorithms of graph theory. The p-median problem, similar to the Weber problem in a plane, finds the locations of p points on a network to minimize the total of demand-weighted distances to the nearest facility. ReVelle and Swain (1970) provide a linear programming formulation. In addition, Hakimi (1964) proposes the seminal p-centre problem, which finds the location of p points on a network to minimize the maximum distance from demand to the nearest facility. The important result of Hakimi's theorem is also given, namely that a solution to the p-median problem always exists at nodes of the network in question. A solution to the p-centre problem, however, does not necessarily exist at nodes. Hakimi (1964) relates all these problems to finding locations for switching centres in a communications network, or for a police station in a road network. Of the many algorithms designed for the p-median problem, we mention that provided by Teitz and Bart (1968), itself a prototype for vertex substitution methods. Further milestones in optimization are provided by Kariv and Hakimi (1979a, b) who prove that the p-centre and p-median problems are NP-hard.

For our next location classic, we consider the simple plant location problem (SPLP), which is a close relative of p-median in terms of objective function. The SPLP seeks to find optimal locations for facilities, each supplying a proportion of customer demand. Total costs are minimized, consisting of fixed costs associated with establishing a facility at a particular location, and with unit supply costs, both production and distribution, from plant to customer. Facilities are assumed to have unconstrained capacity and the number of facilities to be located is not specified. The problem has appeared under a variety of different names, including uncapacitated/simple and warehouse/plant/facility/site location. Likewise the identity of the originator of the SPLP is a matter of debate: several authors might lay claim to this title. Early in the field are Kuehn and Hamburger (1963) and Balinski (1966); earlier work by Balinski and Wolfe (1963) seems to have disappeared from view (Krarup and Pruzan, 1983). Applications include location-finding for warehouses (Kuehn and Hamburger, 1963) and factories (Efroymson and Ray, 1966), as well as schools, hospitals and abattoirs, as suggested by Krarup and Pruzan (1983). Kuehn and Hamburger (1963) propose an early greedy heuristic for the problem, while Efroymson and Ray (1966) propose what was then a relatively new branch-and-bound technique for exact solution. However, Erlenkotter's (1978) method using Lagrangean relaxation with solutions to the dual problem achieves significantly quicker results in finding integer solutions, and can itself be said to be a milestone in algorithmic terms. Van Roy and Erlenkotter (1982) propose extensions to this method, for problems where capacity is constrained. Later on, Beasley (1993) develops a framework to solve the p-median, SPLP and capacitated plant location problems, using heuristics based on the Lagrangean relaxation method.

We move on to what are known as covering models, in locational terms, embodying the notion of the demand covered by facilities within a certain distance or time of travel. Toregas *et al* (1971) formulate and provide a solution method for what has become well known and much used in applications: the location set covering problem (LSCP). The location of facilities for emergency services inspires the problem of finding the minimum number of points that cover all other network nodes within a specified distance. Building on Hakimi's (1965) set covering work, Toregas *et al* supply the necessary constraints for the mathematical programming formulation.

Church and ReVelle (1974) take the set covering problem an important step further with the maximal covering location problem (MCLP), itself a basis for much further research. The problem finds the optimal locations for a given number of facilities by maximizing the population covered within a specified service distance. Both algorithmic and linear programming approaches are used to find solutions, with branch and bound and inspection techniques used to find integer solutions where necessary.

Another fundamental location problem with covering implications is the QAP, so-called because of the quadratic nature of a formulation of its objective function (Lawler, 1963). A number, N, of facilities are located in the same number, N, of sites, so that the total cost of moving materials among them is minimized. The cost of moving materials between any two locations is obtained by multiplying a weight or flow by the distance between the locations. The linear version, introduced by Koopmans and Beckmann (1957), is a special case of the well-known Transportation Problem. A variety of practical applications include facilities layout (Elshafei, 1977) and the placement of electrical components (Miranda *et al*, 2005). This NP-hard problem has generated much subsequent research interest and is still considered intractable at any considerable size. A recent survey is given by Loiola *et al* (2007).

Our final classical model has a completely different methodology from those described previously. As an approach to the location of emergency service vehicles, Larson's (1974) elegantly formed hypercube queuing model has become a classic in its own right. A system of N vehicles is modelled as a continuous-time Markov process with exponential service times. The state space is conceived to be an N-dimensional hypercube, where each vertex describes a particular combination of on-call/idle vehicles. Steady state probabilities are determined using an iterative method. A geographical region, divided into 'cells' or 'atoms', is represented by a matrix of inter-cell travel times and calls are assumed to arise in each cell as a Poisson process.

Period III: Fruitfulness with new models and applications

We time the third period approximately from the 1980s, soon after the first of the triennial International Symposium on Locational Decisions (ISOLDE) conferences in Banff, Canada, in 1978, bringing about the growth of a strong world wide research community in location analysis and related disciplines. In these gatherings, contributors from fields including operational research, management science, economics, engineering and geography continue to be inspired to develop new models and find new applications for the theory of location.

The 1980s and 1990s see research in locational analysis extended into other disciplines, with fruitful results in terms of new modelling and applications. This creativity continues to the present day.

We look at the new modelling areas of competitive location, location of extensive facilities, stochastic location, location-routing, hub location and flow interception. As new applications in this period, we focus on the areas of emergency service planning, environmental applications, including obnoxious facilities, and the combination of location with supply chain management (SCM). The astute reader will notice some instances of early research going back into our previous periods, particularly when other subject disciplines are concerned. We feel, however, that the major growth of location applications has occurred in this period since 1980.

Competitive location models

Competitive location is rooted in the work of Hotelling (1929), as described above, with considerations of likely producer and consumer behaviour. The field of competitive location remained in the hands of economists for a long time: Slater (1975) appears to be the first to have located competing facilities on a network. Hakimi (1983) firmly embeds competitive models within location theory. Much of the work in this area is centred on the determination of Nash and Stackelberg equilibria. A taxonomy and bibliography of the field is provided in Eiselt *et al* (1993). For a further expository paper, see Eiselt and Laporte (1996): most of the results in this area assume a discrete location space or a network. Continuous competitive location models are recently proposed by Dasci and Laporte (2005).

Location of extensive facilities (including network design)

A facility is termed extensive if, in comparison with its surroundings, it is too large to be considered as a point. Such models have frequently been applied in network design situations (Slater, 1982; Labbé *et al*, 1998). Mesa and Boffey (1998) provide a classification system, including problems for routes for transporting hazardous materials. Laporte *et al* (2000) review optimization methods used in planning the alignment and stations of rapid transit systems. A recent example is given by Brimberg *et al* (2007) who address the problem of locating a circle on a sphere, such that distance from existing facilities is minimized. This model could be of use in locating large linear structures such as pipelines on the earth's surface. Laporte and Rodríguez-Martín (2007) review location problems whose solution is a cycle, including variations on the Travelling Salesman Problem, dial-a-ride routes and the Orienteering Problem, which maximizes profit collected *en route* for a limited travel cost.

Stochastic location

Stochastic location models arise when some of the problem data are known only in a probabilistic way. Fixed rather than mobile servers are highlighted here: we discuss in a separate section some of the considerable amount of research that has been applied to the stochastic location of emergency service vehicles.

Several stochastic location–allocation problems have been investigated, of which an early example is Williams (1963). Berman *et al* (1985) investigate problems where arrivals at facilities are random and the effect of congestion must be taken into account. Logendran and Terrell (1988) consider an uncapacitated location–allocation problem with price-sensitive stochastic demands. Carrizosa *et al* (1995) model the location–allocation problem where both customers and facilities are continuously located within regions according to some probability distribution. A generalized formulation is used, which may be applied to a wide range of problems. Marianov and Serra (1998, 2001) consider the location of fixed servers such as primary healthcare centres, banks or distribution centres where congestion exists, including hierarchical situations with referral. Both location set covering and maximal covering models are reformulated to account for congestion.

Berman and Krass (2002) present a general class of 'location problems with stochastic demand and congestion'. Berman *et al* (2003) model the location of a fixed number of facilities on a network, where a probability function describes whether a facility is unable to provide satisfactory service to a customer. Wang *et al* (2003) present models for fixed service facilities such as servers in communications networks or cash point machines, which are congested by stochastic demand originating from nearby locations. Zhou and Liu (2003) propose stochastic programming models for capacitated location–allocation problems. Harper *et al* (2005) develop a generic simulation framework for stochastic location–allocation problems and demonstrate the tool for planning hospital and dental services in the United Kingdom. Surveys of stochastic location models are to be found in Louveaux (1993), and in Snyder (2006).

Location-routing

A combination of location analysis with the well-researched field of vehicle routing problems produces another new area of modelling, location-routing. The location of vehicle bases together with routes for deliveries to clients underlies the problems: it is the interrelationship between the location and routing aspects that gives particular challenges. Webb (1968) is one of the first to recognize the interest of integrating location and routing decisions. Exact models are presented from mid-1980s, for example by Laporte *et al* (1986), with a survey by Laporte (1988). Objectives frequently minimize total travel distance, and uncertainty may be introduced as to whether or not a particular client requires servicing on any given day's route. Variations on the problems include whether the fleet of vehicles is homogenous or heterogenous and whether there are multiple depots or a single depot.

Application areas have included food and drink distribution in the United Kingdom (Watson-Gandy and Dohrn, 1973), medical evacuation in the US Air Force (Chan *et al*, 2001) and parcel delivery in Austria (Wasner and Zäpfel, 2004).

Albareda-Sambola *et al* (2007) study location-routing in a stochastic context. A thorough recent survey of the available literature is given by Nagy and Salhi (2007). Heuristic solutions for these NP-hard problems are classified as clustering-based, iterative or hierarchical.

Hub location

During the last two decades, the growth of hub networks in telecommunications and transportation has engendered a similar growth in design of networks and hub locational analysis. In such location problems, hubs act as concentrators or switching points of traffic, whether for airline passengers, packets in data switching systems or postal transport and deliveries. The flows between origins and destinations provide the modelling basis for this class of problem. Goldman (1969) extends Hakimi's (1964, 1965) network results to produce what is essentially a hub median problem. However, it is O'Kelly (1986a, 1987) who sows the seeds of hub locational analysis, applied to internal US passengers flights. Models are formulated to find the best locations for connecting terminals, minimizing total costs of interactions. Both single-hub and dual-hub systems are considered, with candidate facility locations both in continuous space and at discrete sites.

Research into hub locations in the 1990s has brought comparisons between the classical models for location, such as p-median, SPLP, p-centre and covering models, and their hub location equivalents (Campbell, 1994). Ernst and Krishnamoorthy (1996, 1998) give a generalized approach to formulating hub location problems based on arc flows. Variations on the theme of hub location include direct origin–destination movements without passing through hubs (Aykin, 1994) and congestion (O'Kelly, 1986b). Integrated network and hub location design (such as Bryan, 1998) brings more reality to early models, and level of service considered, as measured by several indicators such as numbers of stops at hubs and path lengths (O'Kelly, 1998). For further reading and a taxonomy of hub location problems, the reader is directed to the survey of Campbell *et al* (2002). New hub arc location problems are introduced by Campbell *et al* (2005a, b).

Flow interception

In many location problems, demand is assumed to occur at nodes of a network. An interesting variation is given by problems where demand is represented by flows of vehicles or pedestrians passing along network links. Applications could include cashpoint machines, petrol stations, drive-through restaurants and walk-in health centres. The main purpose of travel is generally for reasons other than to obtain the service, for example the journey to work might include use of a number of facilities *en route*. However, changes in routes may occur to use the facility. Objectives in this class of problem are to 'capture' maximal flows of travellers, rather than to minimize distances travelled. 'Unwelcome' facility location includes location of radar speed traps. Such problems are first introduced by Hodgson (1981). Berman *et al* (1995) present a number of deterministic flow-interception problems, having differing assumptions about minimum flow and capacity. Probabilistic flow models are also considered, using probable travel origins and probabilities of making turning movements.

Gendreau *et al* (2000) present a unified view of flow interception problems defined with a punitive or a preventive objective. Taking the example of locating policemen on a network to intercept drunk drivers, a punitive objective applies if one wants to intercept as many drivers as possible; a preventive objective consists of intercepting drivers early to maximize the reduction of risk on the network. The punitive case is shown to be a special case of the preventive case.

Emergency service vehicles: location and relocation

A prodigious amount of research has been produced in the study of the location of emergency service vehicles. Marianov and ReVelle (1995) point out the range of optimization models in this area, both deterministic and stochastic as well as descriptive, queuing models. Rosing and Hodgson (1996) classify 43 emergency medical service location–allocation studies between 1971 and 1991, mainly with North American applications. We highlight here both modelling and successful applications in this area.

We mention again the classics of Larson's (1974) hypercube model and Toregas *et al*'s (1971) LSCP, inspired by this public sector problem. Chapman and White (1974) represent the first in terms of probabilistic constraints applied to a version of the LSCP, also introducing the notion of the server busy fraction. The study of Mirchandani and Odoni (1979) considers stochastic travel times in the context of the location of emergency facilities. Daskin and Stern (1981) propose the hierarchical objective set-covering formulation (HOSC) with redundant or backup cover of zones, that is, locating more than one vehicle to cover a particular zone. One possible objective minimizes the number of vehicles needed to cover all zones within a specified time; another objective maximizes the number of multiply covered zones. Also with emergency vehicle applications in mind, Daskin's (1983) MEXCLP extends the MCLP with a probabilistic constraint. Brandeau and Larson (1986) apply Larson's (1974) hypercube in an application in Boston, MA. Eaton *et al* (1986) apply the HOSC in Santo Domingo, Dominican Republic, giving a weighting to different areas by population.

Relocation of vehicles to cope with dynamically changing situations gives another fruitful area for research. Repede and Bernardo (1994) propose TIMEXCLP, which extends MEXCLP with stochastic variation in demand. A simulation model is also used to obtain an improved response time in ambulance relocation. Brotcorne *et al* (2003) provide a survey of research into ambulance location and relocation. Andersson and Värbrand (2007) describe a decision support tool developed for ambulance relocation in Sweden, introducing the concept of preparedness, or the ability to serve potential patients.

Environment-related applications: obnoxious facilities and other concerns

Much of the locational analysis into environmental matters has been concerned with the location of facilities that are unpleasant or harmful to the surrounding population. Goldman and Dearing (1975) and Church and Garfinkel (1978) were among the first to consider locations for obnoxious facilities or facilities that communities prefer to keep at arm's length. Church and Garfinkel coin the term 'maxian' for the opposite to a median-type objective, using maximization of total weighted distances instead of minimization.

Developments in the field of obnoxious facilities have been concerned with both location of single facilities and determination of routes for disposal of harmful substances. Such problems, while minimizing costs, have to take minimization of interaction with a neighbourhood into account. Often multiple objectives are appropriate for realism and multiple solutions sought (Boffey and Karkazis, 1993). Batta and Chiu (1988) model routing of obnoxious facilities on a network, using a Euclidean metric to model spread of harmful toxins, excluding wind effects. Boffey and Karkazis (1995) consider the effect of airborne pollutants on the environment when locating processing sites and on routing vehicles transporting obnoxious materials. A Gaussian plume model is described for location of a facility, which emits relatively inert pollutants such as sulphur dioxide. Erkut and Verter (1995) give an overview of approaches to the logistics of transporting hazardous materials. Suggestions are given for measures of equity, or fairness in the spatial distribution of risk, that may be suitable for locating hazardous materials. A generic multi-objective model is presented for locating hazardous facilities, including possible variation in facility size and content. There is also research interest in the location of semi-obnoxious facilities such as airports desirable for convenience in transport but also undesirable in terms of noise and pollutants. Berman and Wang (2008) model semi-obnoxious facilities with an expropriation budget, which compensates nearby residents for the obnoxious effects.

We turn to problems of conservation of the environment. Church *et al* (1998) review location issues in forest management. Classical covering problems have been adapted to cover animal species rather than the human variety, giving the species set cover problem (Underhill, 1994) and the maximal covering species problem (Church *et al*, 1996). ReVelle and

Williams (2002) consider the problems of game reserve design, which provides an interesting combination of discrete location with geographical and ecological considerations. Following on, Önal and Wang (2007) address fragmentation of the habitats of species in reserves. Marianov *et al* (2008) consider the different needs of species in selecting sites and Williams (2008) includes the distances between reserve sites in modelling.

Finally, we give some recent examples of location problems of current environmental interest. Church *et al* (2004) propose the *r*-interdiction median model, which seeks to identify important links and structures in systems that may be subject to disruption through interdiction, that is, natural disaster or man-made attack. Church and Scaparra (2007) extend that modelling to give the possibility of fortification against such incidents. Eiselt (2007) analyses the location of landfill sites in New Brunswick, Canada, using an optimizing model based on hub location principles. Yi and Özdamar (2007) present an integrated location-distribution model for disaster relief activities. The location of sites for emergency centres in regions of safety is combined with relocation of medical staff from neighbouring permanent medical facilities. Cáceres *et al* (2007) model the location of waste pipelines to minimize damage to environmentally sensitive areas such as coral reefs and sandbanks.

Location analysis with SCM

SCM involves a number of decisions in supply, production and distribution, regarding number and location of facilities and network flows. From some early roots, a fruitful collaboration has grown between SCM and locational analysis over the last 40 years. A recent comprehensive review is given by Melo *et al* (2009) with a focus on discrete problems.

The seminal work in dynamic planning of Ballou (1968) makes use of dynamic programming for relocation of warehouses over the planning period. The paper of Warszawski (1973) provides an early work on multi-commodity production problems, with each factory producing a single commodity. Aikens (1985) gives an early review of contributions to this area of research. Klincewicz and Luss (1987) provide an algorithm for the multi-commodity uncapacitated plant location problem, inspired by Erlenkotter (1978). Geoffrion and Powers (1995) consider the areas of unity between location and SCM, including capacity considerations as well as location decisions. Verter and Dasci (2002) present an integrated framework for optimizing location, capacity and technological decisions. Talluri and Baker (2002) take a multi-phase approach to supply chain design, combining game theory concepts with mathematical programming. Melo *et al* (2005)'s modelling framework provides a complex description of supply chain problems, including multi-period aspects of planning.

Given current worldwide company strategies of restructuring on a multi-national basis, global SCM has become a research topic of interest, bringing greater complexity than is found in domestic situations. Verter and Dincer (1995) review models of production and distribution with a special emphasis on global SCM, as do Vidal and Goetschalckx (1997). Daskin *et al* (2002) model the location of distribution centres, taking into account inventory levels and transportation costs. Hinojosa *et al* (2008) consider a dynamic situation of opening and closing facilities to meet customer needs over a stated time horizon.

Reverse logistics, the management of recycled goods, has received attention from location theorists in recent years. Production networks are extended to take account of return flows from customers. An early work in the distribution area is provided by Gottinger (1988) and a review by Fleischmann *et al* (1997). Marín and Pelegrin (1998) model returns made direct to the producing factories. Salema *et al* (2007) give generalized models for material recovery. Srivastava (2008) gives a recent review with a viewpoint of green supply chain logistics in India.

Location analysis for the developing world

Moving to problems encountered in developing world scenarios, Banerji and Fisher (1974) provide an early example of the use of locational techniques in this context. The use of both *p*-median and covering models is suggested for rural development planning in India. A covering model is used by Eaton *et al* (1981) to improve locations of ambulance bases in Colombia. Hodgson (1988) develops a hierarchical model for planning primary healthcare in India.

A comprehensive review of the use of location–allocation models for healthcare facilities in developing countries is provided by Rahman and Smith (2000). Spatial choice-based models are discussed by Yasenovskiy and Hodgson (2007) with application to healthcare facilities in Ghana. Smith *et al* (2009) propose the MNS model for the location of the maximal number of sustainable facilities, inspired by the problem of finding the maximum number of self-financing healthcare workers sustainable in a developing rural region.

Conclusion

We have considered the growth of location theory from its early beginnings, through a period of coming of age, and on to the current period of new models and applications. The reader will note areas of overlap between the new modelling areas and applications. There are links, for example, between environmental concerns and the reverse logistics of SCM, while SCM itself is connected with location-routing. This is indicative of the richness and complexity of current locational research, characteristics likely to continue and intensify.

Adaptations to the problems of today's society will continue to bring new areas for research interest. Developing world problems have received relatively little of the recent attention of locational modellers: surely the radically different contexts of a developing nation can provide a new series of problems.

Addressing problems of current importance, such as the incidence of man-made or natural disaster, there will be scope for new models to provide solutions to new applications.

As we look back over the last decades, we see the rich heritage of locational theory. Research continues to be based on these fundamentals, while new areas continue to come into being. Locational analysis is in very much a healthy state, and will continue to grow in future time periods in a similarly dynamic manner.

Acknowledgements— Thanks are due to two referees for their valuable comments.

References

Aikens CH (1985). Facility location models for distribution planning. *Eur J Opl Res* **22**: 263–279.

Albareda-Sambola M, Fernández E and Laporte G (2007). Heuristic and lower bound for a stochastic location-routing problem. *Eur J Opl Res* **179**: 940–955.

Andersson T and Värbrand P (2007). Decision support tools for ambulance dispatch and relocation. *J Opl Res Soc* **58**: 195–201.

Aykin T (1994). Lagrangian relaxation based approaches to capacitated hub-and-spoke network design problem. *Eur J Opl Res* **79**: 501–523.

Balinski ML (1966). *On Finding Integer Solutions to Linear Programs*. Mathematica: Princeton, NJ.

Balinski ML and Wolfe P (1963). *On Benders decomposition and a plant location problem*. Working paper ARO-27. Mathematica: Princeton, NJ.

Ballou RH (1968). Dynamic warehouse location analysis. *J of Market Res* **5**: 271–276.

Banerji SH and Fisher HB (1974). Hierarchical location analysis for integrated area planning in rural India. *Pap Region Sci Assoc* **33**: 177–194.

Batta R and Chiu SS (1988). Optimal obnoxious paths on a network: Transportation of hazardous materials. *Opns Res* **36**: 84–92.

Beasley JE (1993). Lagrangean heuristics for location problems. *Eur J Opl Res* **65**: 383–399.

Berman O and Krass D (2002). Facility location problems with stochastic demands and congestion. In: Drezner Z and Hamacher H (eds). *Facility Location: Applications and Theory*. Springer-Verlag: Berlin, pp 329–371.

Berman O and Wang Q (2008). Locating a semi-obnoxious facility with expropriation. *Comput Opns Res* **35**: 392–403.

Berman O, Larson RC and Chiu SS (1985). Optimal server location on a network operating as a M/G/1 queue. *Opns Res* **33**: 746–770.

Berman O, Hodgson MJ and Krass D (1995). Flow-interception problems. In: Drezner Z (ed). *Facility Location: A survey of Applications and Methods*. Springer: New York, pp 389–426.

Berman O, Drezner Z and Wesolowsky GO (2003). Locating service facilities whose reliability is distance dependent. *Comput Opns Res* **30**: 1683–1695.

Boffey TB and Karkazis J (1993). Models and methods for location and routing decisions relating to hazardous materials. *Stud Location Anal* **5**: 149–166.

Boffey TB and Karkazis J (1995). Location, routing and the environment. In: Drezner Z (ed). *Facility Location: A Survey of Applications and Methods*. Springer: New York, pp 453–466.

Brandeau ML and Larson RC (1986). Extending and applying the hypercube model to deploy ambulances in Boston. In: Swersey AJ and Ignall EJ (Eds). *Delivery of Urban Service: With a View Towards Applications in Management Science and Operations Research. TIMS Studies in the Management Sciences*, Vol. 22. North-Holland: Amsterdam, pp 121–153.

Brotcorne L, Laporte G and Semet F (2003). Ambulance location and relocation models. *Eur J Opl Res* **147**: 451–468.

Brimberg J, Juel H and Schöbel A (2007). Locating a circle on a sphere. *Opns Res* **55**: 782–791.

Bryan DL (1998). Extensions to the hub location problem: Formulations and numerical examples. *Geogr Anal* **30**: 315–330.

Cáceres T, Mesa JA and Ortega FA (2007). Locating waste pipelines to minimize their impact on marine environment. *Eur J Opl Res* **179**: 1143–1159.

Campbell JF (1994). Integer programming of formulations of discrete hub location problems. *Eur J Opl Res* **72**: 387–405.

Campbell JF, Ernst AT and Krishnamoorthy M (2002). Hub location problems. In: Drezner Z and Hamacher H (eds). *Facility Location: Applications and Theory*. Springer-Verlag: Berlin, pp 373–407.

Campbell JF, Ernst AT and Krishnamoorthy M (2005a). Hub arc location problems: Part I—Introduction and results. *Mngt Sci* **51**: 1540–1555.

Campbell JF, Ernst AT and Krishnamoorthy M (2005b). Hub arc location problems: Part II—Formulations and optimal algorithms. *Mngt Sci* **51**: 1556–1571.

Carrizosa E, Conde E, Munoz-Marquez M and Puerto J (1995). The generalized Weber problem with expected distances. *Rech Opér* **29**: 35–57.

Chan Y, Carter WB and Burnes MD (2001). A multiple-depot, multiple-vehicle, location-routing problem with stochastically processed demands. *Comput Opns Res* **28**: 803–826.

Chapman SC and White JA (1974). Probabilistic formulations of emergency service facilities location problems. Reprint series 7407, IEOR Dept., VPI&SV.

Church RL and Garfinkel RS (1978). Locating an obnoxious facility on a network. *Transport Sci* **12**: 107–118.

Church RL and ReVelle CS (1974). The maximal covering location problem. *Pap Region Sci Assoc* **32**: 101–118.

Church RL and Scaparra MP (2007). Protecting critical assets: The r-interdiction median problem with fortification. *Geog Anal* **39**: 129–146.

Church RL, Stoms DM and Davis FW (1996). Reserve selection as a maximal covering location problem. *Biol Conserv* **76**: 105–112.

Church RL, Murray AT and Weintraub A (1998). Locational issues in forest management. *Locat Sci* **6**: 137–153.

Church RL, Scaparra MP and Middleton R (2004). Identifying critical infrastructure: The median and covering facility interdiction problems. *Ann Assoc Am Geogr* **94**: 491–502.

Cooper L (1963). Location–allocation problems. *Opns Res* **11**: 331–343.

Cooper L (1964). Heuristic methods for location-allocation problems. *SIAM Rev* **6**: 37–53.

Dasci A and Laporte G (2005). A continuous model for multi-store competitive location. *Opns Res* **53**: 263–280.

Daskin MS (1983). A maximum expected covering location model: Formulation, properties and heuristic solution. *Transport Sci* **17**: 48–70.

Daskin MS (2008). What you should know about location modeling. *Naval Res Logis* **55**: 283–294.

Daskin MS and Stern E (1981). A hierarchical objective set covering model for emergency medical service deployment. *Transport Sci* **25**: 137–152.

Daskin MS, Coullard C and Shen Z-JM (2002). An inventory-location model: Formulation, solution algorithm and computational results. *Ann Opns Res* **110**: 83–106.

Delaunay B (1934). Sur la sphère vide. *Izvestia Akademii Nauk SSSR, Otdelenie Matematicheskikh i Estestvennykh Nauk* **7**: 793–800.

Drezner Z, Klamroth K, Schöbel A and Wesolowsky GO (2002). The Weber problem. In: Drezner Z and Hamacher H (eds). *Facility*

Location: Applications and Theory. Springer-Verlag: Berlin, pp 1–36.

Eaton DJ, Church RL, Bennett VL and Hamon BL (1981). On deployment of health resources in rural Valle del Cauca, Colombia. *TIMS Stud Mngt Sci* **17**: 331–359.

Eaton DJ, Sánchez HML, Lantigua RR and Morgan J (1986). Determining ambulance deployment in Santo Domingo, Dominican Republic. *J Opl Res Soc* **37**: 113–126.

Efroymson MA and Ray TL (1966). A branch-bound algorithm for plant location. *Opns Res* **14**: 361–368.

Eiselt HA (2007). Locating landfills—Optimization vs. reality. *Eur J Opl Res* **179**: 1040–1049.

Eiselt HA and Laporte G (1996). Sequential location problems. *Eur J Opl Res* **96**: 217–231.

Eiselt HA, Laporte G and Thisse J-F (1993). Competitive location models: A framework and bibliography. *Transport Sci* **27**: 44–54.

Elshafei AN (1977). Hospital layout as a quadratic assignment problem. *Opl Res Quart* **28**: 167–179.

Erkut E and Verter V (1995). Hazardous materials logistics. In: Drezner Z (ed). *Facility Location: A Survey of Applications and Methods*. Springer: New York, pp 467–506.

Erlenkotter D (1978). A dual-based procedure for uncapacitated facility location. *Opns Res* **26**: 992–1009.

Ernst AT and Krishnamoorthy M (1996). An exact solution approach based on shortest-paths for p-hub median problems. *INFORMS J Comput* **10**: 149–162.

Ernst AT and Krishnamoorthy M (1998). Exact and heuristic algorithms for the uncapacitated multiple allocation p-hub median problem. *Eur J Opl Res* **104**: 100–112.

Fleischmann M, Bloemhof-Ruwaard JM, Dekker R, van der Laan E, van Nunen JAEE and Van Wassenhove LN (1997). Quantitative models for reverse logistics: A review. *Eur J Opl Res* **103**: 1–17.

Geoffrion AM and Powers RF (1995). Twenty years of strategic distribution system design: An evolutionary perspective. *Interfaces* **25**(5): 105–127.

Gendreau M, Laporte G and Parent I (2000). Heuristics for the location of inspection stations on a network. *Naval Res Logis* **47**: 287–303.

Goldman AJ (1969). Optimal locations for centers in a network. *Transport Sci* **3**: 352–360.

Goldman AJ and Dearing PM (1975). Concepts of optimal location for partially noxious facilities. *ORSA Bull* **23**(1): B-31.

Gottinger HW (1988). A computational model for solid waste management with application. *Eur J Opl Res* **35**: 350–364.

Hakimi SL (1964). Optimum locations of switching centers and the absolute centers and medians of a graph. *Opns Res* **12**: 445–450.

Hakimi SL (1965). Optimum distribution of switching centers in a communication network and some related graph theoretic problems. *Opns Res* **13**: 462–475.

Hakimi SL (1983). On locating new facilities in a competitive environment. *Eur J Opl Res* **12**: 29–55.

Harper PR, Shahani AK, Gallagher J and Bowie C (2005). Planning health services with explicit geographical considerations: A stochastic location–allocation approach. *Omega* **33**: 141–152.

Hinojosa Y, Kalcsics J, Nickel S, Puerto J and Velten S (2008). Dynamic supply chain design with inventory. *Comput Opns Res* **35**: 373–391.

Hodgson MJ (1981). The location of public facilities intermediate to the journey to work. *Eur J Opl Res* **6**: 199–204.

Hodgson MJ (1988). An hierarchical location-allocation model for primary health care delivery in a developing area. *Soc Sci Med* **26**: 153–161.

Hotelling H (1929). Stability in competition. *Econ J* **39**: 41–57.

Isard W (1969). *General Theory: Social, Political, Economic, and Regional, with Particular Reference to Decision-making Analysis*. MIT Press: Cambridge, MA.

Klincewicz JG and Luss H (1987). A dual-based algorithm for multi-product uncapacitated facility location. *Transport Sci* **21**: 198–206.

Kariv O and Hakimi SL (1979a). An algorithmic approach to network location problems. Part I: The P-centers. *SIAM J Appl Math* **37**: 515–538.

Kariv O and Hakimi SL (1979b). An algorithmic approach to network location problems. Part II: The P-median. *SIAM J Appl Math* **37**: 539–560.

Koopmans TC and Beckmann MJ (1957). Assignment problems and the location of economic activities. *Econometrica* **25**: 52–76.

Krarup J and Pruzan PM (1983). The simple plant location problem: Survey and synthesis. *Eur J Opl Res* **12**: 36–81.

Kuehn AA and Hamburger MJ (1963). A heuristic program for locating warehouses. *Mngt Sci* **9**: 643–666.

Kuhn HW (1967). On a pair of dual nonlinear programs. In: Abadie J (ed). *Nonlinear Programming*. North-Holland: Amsterdam, pp 37–54.

Labbé M, Laporte G and Rodríguez-Martín I (1998). Path, Tree and Cycle Location. In: Crainic TG and Laporte G (eds). *Fleet Management and Logistics*. Kluwer: Boston, pp 187–204.

Laporte G (1988). Location-Routing problems. In: Golden BL and Assad AA (eds). *Vehicle Routing: Methods and Studies*. North-Holland: Amsterdam, pp 163–198.

Laporte G and Rodríguez-Martín I (2007). Locating a cycle in a transportation or a telecommunications network. *Networks* **50**: 92–108.

Laporte G, Mesa JA and Ortega FA (2000). Optimization methods for the planning of rapid transit systems. *Eur J Opl Res* **122**: 1–10.

Laporte G, Nobert Y and Arpin D (1986). An exact algorithm for solving a capacitated location-routing problem. *Ann Opns Res* **6**: 293–310.

Larson RC (1974). A hypercube queuing model for facility location and redistricting in urban emergency services. *Comput Ops Res* **1**: 67–95.

Lawler EL (1963). The quadratic assignment problem. *Mngt Sci* **9**: 586–599.

Logendran R and Terrell MP (1988). Uncapacitated plant location–allocation problems with price sensitive stochastic demands. *Comput Opns Res* **15**: 189–198.

Loiola EM, Maia de Abreu NM, Boaventura-Netto PO, Hahn P and Querido T (2007). A survey for the quadratic assignment problem. *Eur J Opl Res* **176**: 657–690.

Louveaux FV (1993). Stochastic location analysis. *Locat Sci* **1**: 127–154.

Maranzana FE (1964). On the location of supply points to minimize transport costs. *Opl Res Quart* **15**: 261–270.

Marianov V and ReVelle CS (1995). Siting emergency services. In: Drezner Z (ed). *Facility Location: A Survey of Applications and Methods*. Springer: New York, pp 199–223.

Marianov V and Serra D (1998). Probabilistic, maximal covering location–allocation models for congested systems. *J Region Sci* **38**: 401–424.

Marianov V and Serra D (2001). Hierarchical location–allocation models for congested systems. *Eur J Opl Res* **135**: 195–208.

Marianov V, ReVelle CS and Snyder S (2008). Selecting compact habitat reserves for species with differential habitat size needs. *Comput Opns Res* **35**: 475–487.

Marín A and Pelegrin B (1998). The return plant location problem: Modelling and resolution. *Eur J Opl Res* **104**: 375–392.

Melo MT, Nickel S and Saldanha da Gama F (2005). Dynamic multi-commodity capacitated facility location: A mathematical modeling framework for strategic supply chain planning. *Comput Opns Res* **33**: 181–208.

Melo MT, Nickel S and Saldanha da Gama F (2009). Facility location and supply chain management–A review. *Eur J Opl Res*, **196**: 401–412.

Mesa JA and Boffey TB (1998). A review of extensive facility location in networks. *Eur J Opl Res* **95**: 592–603.
Miranda G, Luna HPL, Mateus GR and Ferreira RPM (2005). A performance guarantee heuristic for electronic components placement problems including thermal effects. *Comput Ops Res* **32**: 2937–2957.
Mirchandani PB and Odoni AR (1979). Location of medians on stochastic networks. *Transport Sci* **13**: 85–97.
Nagy G and Salhi S (2007). Location-routing: Issues, models and methods. *Eur J Opl Res* **177**: 649–672.
O'Kelly ME (1986a). The location of interacting hub facilities. *Transport Sci* **20**: 92–106.
O'Kelly ME (1986b). Activity levels at hub facilities in interacting networks. *Geogr Anal* **18**: 343–353.
O'Kelly ME (1987). A quadratic integer program for the location of interacting hub facilities. *Eur J Opl Res* **32**: 393–404.
O'Kelly ME (1998). On the allocation of a subset of nodes to a mini-hub in a package delivery network. Papers in Regional Science. *J RSAI* **77**: 77–99.
Önal H and Wang Y (2007). A graph theory approach for designing conservation reserve networks with minimal fragmentation. *Networks* **51**: 142–152.
Rahman S and Smith DK (2000). Use of location–allocation models in health service development planning in developing nations. *Eur J Opl Res* **123**: 437–452.
Repede JF and Bernardo JJ (1994). Developing and validating a decision support system for locating emergency medical vehicles in Louisville, Kentucky. *Eur J Opl Res* **75**: 567–581.
ReVelle CS and Eiselt HA (2005). Location analysis: A synthesis and survey. *Eur J Opl Res* **165**: 1–19.
ReVelle CS and Swain R (1970). Central facilities location. *Geogr Anal* **2**: 30–42.
ReVelle CS and Williams JC (2002). Reserve design and facility siting. In: Drezner Z and Hamacher H (eds). *Facility Location: Applications and Theory*. Springer-Verlag: Berlin, pp 307–328.
ReVelle CS, Eiselt HA and Daskin MS (2008). A bibliography for some fundamental problem categories in discrete location science. *Eur J Opl Res* **184**: 817–848.
Rosing KE and Hodgson MJ (1996). A systematic classification of applications of location-allocation models. *Belgian J Opns Res, Stat Comput Sci* **36**: 77–108.
Salema MIG, Barbosa-Povoa AP and Novais AQ (2007). An optimization model for the design of a capacitated multi-product reverse logistics network with uncertainty. *Eur J Opl Res* **179**: 1063–1077.
Slater PJ (1975). Maximum facility location. *J Res NBS B Math Sci* **79**: 107–115.
Slater PJ (1982). Locating central paths in a graph. *Transport Sci* **16**: 1–18.
Smith HK, Harper PR, Potts CN and Thyle A (2009). Planning sustainable community health schemes in rural areas of developing countries. *Eur J Opl Res* **193**:768–777.
Snyder LV (2006). Facility location under uncertainty: A review. *IIE Trans* **38**: 547–564.
Srivastava SK (2008). Network design for reverse logistics. *Omega* **36**: 535–548.
Suzuki A and Okabe A (1995). Using Voronoi diagrams. In: Drezner Z (ed). *Facility Location: A Survey of Applications and Methods*. Springer: New York, pp 103–118.
Talluri S and Baker RC (2002). A multi-phase mathematical programming approach for effective supply chain design. *Eur J Opl Res* **141**: 544–558.
Tansel BC, Francis RL and Lowe TJ (1983a). Location on networks: A survey, Parts I and II. *Mngt Sci* **29**: 482–497.
Tansel BC, Francis RL and Lowe TJ (1983b). Location on networks: A survey, Parts I and II. *Mngt Sci* **29**: 498–511.
Teitz MB and Bart P (1968). Heuristic methods for estimating the generalized vertex median of a weighted graph. *Opns Res* **16**: 955–961.
Toregas C, Swain R, ReVelle C and Bergman L (1971). The location of emergency service facilities. *Opns Res* **19**: 1363–1373.
Underhill LG (1994). Optimal and suboptimal reserve selection algorithms. *Biol Conserv* **70**: 85–87.
Verter V and Dasci A (2002). The plant location and flexible technology acquisition problem. *Eur J Opl Res* **136**: 366–382.
Verter V and Dincer MC (1995). Global manufacturing strategy. In: Drezner Z (ed). *Facility Location: A Survey of Applications and Methods*. Springer-Verlag: New York, pp 263–282.
Van Roy T and Erlenkotter D (1982). A dual-based procedure for dynamic facility location. *Mngt Sci* **28**: 1091–1195.
Vidal CJ and Goetschalckx M (1997). Strategic production-distribution models: A critical review with emphasis on global supply chain models. *Eur J Opl Res* **98**: 1–18.
Voronoi G (1907). Nouvelles applications des paramètres continus à la théorie des formes quadratiques. *J Reine Angew Math* **133**: 97–178.
Wang Q, Batta R and Rump CM (2003). Facility location models for immobile servers with stochastic demand. *Naval Res Logis* **51**: 137–152.
Warszawski A (1973). Multidimensional location problems. *Opl Res Quart* **24**: 165–179.
Wasner M and Zäpfel G (2004). An integrated multi-depot hub-location vehicle routing model for network planning of parcel service. *Int J Prod Econ* **90**: 403–419.
Watson-Gandy CDT and Dohrn PJ (1973). Depot location with van salesmen—A practical approach. *Omega* **1**: 321–329.
Webb MHJ (1968). Cost functions in the location of depots for multi-delivery journeys. *Opl Res Quart* **19**: 311–320.
Weber A (1909). *Über den Standort der Industrien*. Mohr: Tübingen, Germany.
Weiszfeld E (1937). Sur le point pour lequel la somme des distances de *n* points donnés est minimum. *Tôhoko Math J* **43**: 355–386.
Williams AC (1963). A stochastic transportation problem. *Opns Res* **11**: 759–770.
Williams JC (2008). Optimal reserve site selection with distance requirements. *Comput Opns Res* **35**: 475–487.
Yasenovskiy VS and Hodgson MJ (2007). Hierarchical location–allocation with spatial choice interaction modeling. *Ann Assoc Am Geogr* **97**: 496–511.
Yi W and Özdamar L (2007). A dynamic logistics coordination model for evacuation and support in disaster response activities. *Eur J Opl Res* **179**: 1177–1193.
Zhou J and Liu B (2003). New stochastic models for capacitated location–allocation problem. *Comput Indust Eng* **45**: 111–125.

Received July 2008;
accepted October 2008 after one revision