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Fifty years of probabilistic decision analysis: a view from the UK

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In 1959, LJ Savage attended a statistics seminar held in his honour at the University of London, confronting those present with a radically different approach to reasoning about uncertainty. Britain was well placed to respond to Savage, as very similar ideas had been laid out in Britain a full generation earlier, and in the next few decades, British and British-based practitioners and researchers championed a collection of techniques for thinking quantitatively about uncertainty (which we call 'Probabilistic Decision Analysis'), developing practice, and contributing to theoretic knowledge about the underlying psychology and mathematics. This effectively turned a collection of purely theoretical ideas into a practical modelling technology. In the first decade of the 21st century, some 50 years on, these ideas have made a noticeable influence on practice and thinking in various domains, but numerous challenges still remain.

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Introduction

This paper presents a short history of probabilistic decision analysis (PDA) focussing particularly on the UK, covering both practical applications and significant contributions to underlying concepts and mathematics. We restrict the scope of the present review to focus on workers who are either British nationals or have been UK-based for much of their careers, and on those pillars of the UK academic landscape, the *Journal of the Operational Research Society* and its previous incarnation, *OR Quarterly*, and the *Journal of the Royal Statistical Society* in its various manifestations. The reason for this choice is partly for practicability—a review which attempted to cover the entire field internationally would inevitably be even more partial and even less comprehensive than the current paper. But a second motivation is to fill a gap: an independent British tradition of PDA exists which has been largely neglected in the mostly US-focussed retrospectives (eg Raiffa (2002) and the tribute volumes for Ward Edwards (Shanteau *et al*, 1999) and Howard Raiffa (Zeckhauser *et al*, 1996)).

We should perhaps begin by acknowledging that the term *Probabilistic Decision Analysis* is not in widespread use. We take it to refer to the use of quantitative methods, in particular probability theory, to support decision making under conditions of uncertainty (in explicit contrast to decision making under multiple objectives). In our view, without quantifying uncertainty it is impossible to be sure what one means either in communications with others, or indeed internally, within oneself. We focus on the quantification of uncertainty through the use of probability theory because, although probability theory is not the only formalization of reasoning under uncertainty (eg Schum, 1994), it is by far the best explored formal framework, and is almost certainly the one most familiar to the reader.

We have deliberately avoided the terms *Statistical Decision Theory* and *Probabilistic Risk Analysis* for the purposes of the present review. The former seems inappropriate for a review of an eminently practical modelling technology, especially one with particular applicability to situations where directly relevant data may be scarce or unavailable. The latter omits to mention the critical concept of 'decision'. It is true that there do exist situations, which can be thought of as 'pure' inference problems with no decision component, for example in basic scientific research (Jaynes and Bretthorst, 2003; Howson and Urbach, 2005), but in OR, the question of what the manager or policy maker should do is highly relevant (Blackett, 1950).

Originating ideas

Throughout its history, probability theory has had a close relationship with questions of how to decide, being stimulated by practical problems in gambling and insurance (David, 1962; Hacking, 1984; Bernstein, 1996). But the notion of how best to think formally about decision making under uncertainty became a widespread preoccupation in the early years of OR—see for instance Thrall *et al* (1954). As much of

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the relevant work took place in the USA, this does not fall squarely within the scope of the current review. However, one milestone in this intellectual tradition is worth dealing with in depth: the publication in 1954 of Savage's *The Foundations of Statistics* (Savage, 1954). In his address to the Joint Statistics seminar of Birkbeck and Imperial colleges during a 1959 visit to London, Savage confronted many of the UK's leading statisticians with a radically different approach to reasoning about uncertainty (Savage *et al*, 1962). We outline Savage's ideas by quoting from a 1958 review by someone who was to become a vocal British champion of Savage's ideas in the decades ahead:

> [Savage] starts from a situation in which the world is in a number of possible 'states' and the statistician has to make one among a number of possible 'decisions' . . . Savage proposes certain rules and then says that a man is 'rational' if he obeys these rules in his decision making. . . Having defined his 'rational' man, Savage spends a large part of his book proving the following result: a rational man always acts as if he had a probability distribution over the states of the world and a utility function describing the utility to him of each decision in each state; he chooses his decision by maximising the expected utility. (Lindley, 1958, pp 191–192)

Critically for our future discussion, the rules which Savage's decision maker is presumed to obey centre on his preferences over alternative actions: it is assumed in various ways that these preferences are well-defined; that his beliefs about the relative chances of the various states are consistent with those preferences; and that his preferences about the consequences realized by a certain action in a certain state are consistent with his preferences over actions.

Given these assumptions expressed in suitable formalism plus some auxiliary mathematical machinery, it is possible to deduce the following result:

Theorem 1 *There exist unique probabilities associated with each state and unique utilities associated with each consequence such that an action a is preferred to an action b if and only if the expected utility* (*the sum of the products of the utilities associated with the consequence in each state and the probability of that state*) *of action a is greater than that of action b.*

It is ironic that Savage's ideas came as such a shock to the British statistical establishment of 1954, because very similar ideas had been laid out in Britain a full generation earlier. In the 1920s, with the laws of probability well-established, and the development of statistics as a practical technology with widespread application well under way, there was considerable dispute about the *meaning* of probability (Howie, 2002). Could probability represent a *degree of belief*, and if so, what did it mean to have a degree of belief? In a paper written in 1926, entitled *Truth and Probability*, FP Ramsey developed a formal theory based around a surprising answer to this question:

> When we seek to know what is the difference between believing more firmly, and believing less firmly, we can no longer regard it as consisting in having more or less of certain observable feelings; at least I personally cannot recognise any such feelings. The difference seems to me to lie in how far we should act on these beliefs . . . our judgement about the strength of our belief is really about how we should act in hypothetical circumstances. (Ramsey, 1978, pp 72–73)

In order to make this concrete, Ramsey suggests that people make decisions according to the mathematical expectations of 'goods or bads'—what we would nowadays call utility. He then argues that his intuition about belief acquiring meaning through its relation to action can be used as the basis for a measurement:

> I am at a cross-roads and do not know the way; but I rather think that one of the two ways is right. I propose therefore to go that way, but to keep my eyes open for someone to ask; if now I see someone half a mile away over the fields, whether I turn aside to ask him will depend on the relative inconvenience of going out of my way to cross the fields or of continuing on the wrong road if it is the wrong road. But it will also depend on how confident I am that I am right; and clearly the more confident I am of this the less distance I should be willing to go from the road to check my opinion. (*ibid*, pp 76–77)

Ramsey's decision problem is depicted in Table 1. The actions are to *continue* and *to go out of your way to get directions*, and the states are *on the right road* and *on the wrong road*. Ramsey defines the utility of two consequences: *arriving at the right destination* which is given utility r; and *arriving at the wrong destination*, which is given utility w, but there is a second issue to be taken into account, as if he goes out of his way, he will certainly arrive at the right destination, but only after a diversion. Ramsey proposes a disutility function, $f(\bullet)$, which takes as its argument the distance x to be travelled away from the road. He supposes this combines additively with r. Obviously there is only a decision to be made as long as $f(x) < r - w$, so interest can be restricted to values of x such that this inequality is satisfied. Let p represent the subjective probability that Ramsey is on the right road. The expected utility of continuing is $w + p(r - w)$ and the expected utility of turning aside to ask directions is $r - f(x)$, as a function x of the distance Ramsey has to go out of his way (these utilities suppose that there is no cost to returning up the road once he learns he is in error, so we presume that Ramsey encounters his decision problem before having continued too far from the crossroads).

Now, if Ramsey finds a distance x^o such that he is prepared to go x^o out of his way and no further, x^o must equate the utilities of the two actions:

$$w + p(r - w) = r - f(x^o)$$

Table 1 Ramsey's decision problem

	States	
	On the right road	*On the wrong road*
Actions		
Continue	Right destination	Wrong destination
Go out of your way to get directions	Right destination, after diversion	Right destination, after diversion

Solving for p yields,

$$p = 1 - \frac{f(x^o)}{r - w}$$

Thus, the equilibrating distance x^o can be used to determine a measure for that elusive concept, degree of belief. Here we see a critical part of Savage's framework, namely the idea of using preferences over action to derive a quantitative (though subjective) probability.

Despite the depth and originality of Ramsey's ideas, *Truth and Probability* was ahead of its time: Savage himself notes that 'Ramsey's essays, though now much appreciated, seem to have had relatively little influence' (Savage, 1954, p 60), and Ramsey unfortunately died at the age of 26. Harold Jeffreys, a geophysicist, knew Ramsey in the 1920s, but Jeffreys' pioneering work on 'inverse probability' (the Bayesian moniker came later) was more focussed on the problem of inference than decision making (Jeffreys, 1939). Before Savage's generation could rediscover Ramsey for themselves, an alternative and competing account of the relationship between probability and decision had emerged—an account forever associated with the names of John von Neumann and Oskar Morgenstern.

To give a sense of the style of the von Neumann–Morgenstern account, we informally sketch the axiomatic treatment of utility developed by Luce and Raiffa (1957), who suppose that we have a decision maker who has to choose between a number of lotteries, each with specified probabilities of realizing particular consequences. Also on the table are second-order lotteries, in which the prizes are themselves lotteries, third-order lotteries, and so on. Our decision maker has preference orderings over the consequences and also over the lotteries themselves, but is only interested in the outcomes of the lotteries: he does not feel particularly strongly about the process of gambling itself, or the number of times he has to gamble to get to a consequence. Given a suitable set of axioms, it is possible to deduce the following result:

Theorem 2 *There exist unique utility scores associated with each consequence such that a lottery a is preferred to a lottery b, if and only, if the expected utility of a lottery a is greater than the expected utility of a lottery b.*

How might we know what these utilities are? The proof of Theorem 2 gives an indication of how an analyst might engage with a decision maker to elicit his utilities. We demonstrate this with an example. Suppose you have contracted a rare eye disease. If left untreated, this disease will eventually render you *blind* (a consequence). Your ophthalmologist tells you that there is an *operation* (ie a lottery) which, if successful, will *restore your vision* completely (a second, happier, consequence), but if not, you will *die* on the operating table (your least favourite consequence). This decision is represented using the conventional decision tree formalism in Figure 1; the probability of a successful operation is p.

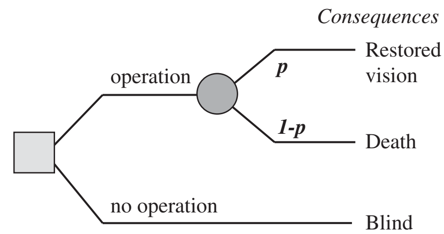

Figure 1 Constructing von Neumann–Morgenstern utilities.

This cover story can be used to construct a von Neumann–Morgenstern utility of the state of being blind. To do this, we fix a scale by calling the utility of being dead 0, the utility of being in full health, 1, and the utility of being blind, u. We find u by varying the probability p until we find a value that makes us indifferent between the upper and lower branches (and call this p^o). Once we have $p = p^o$, we know that the expected utilities of the upper and lower branch of the decision tree must be equal. The expected utility of the upper branch of the decision tree is $(p^o \times 1) + ((1 - p^o) \times 0) = p^o$, and the (expected) utility of the lower branch is u and so $u = p^o$: we have found our utility for the state of being blind.

As can be seen from the above example, the Ramsey view of the relation between probability and preference is in some sense the inverse of the von Neumann–Morgenstern view. In the former, we assume from the start quantitative values over the objects of choice, and from this construct our measure of probability; in the latter, probabilities are assumed to be known and these are used to construct a measure of preference. That thinkers as distinguished as Ramsey and von Neumann could alight on different primitive elements in their axiomatizations of decision making under uncertainty highlights for us that neither are concepts which naturally pre-exist in the mind of decision makers. In turn, this underscores that both probabilities and utilities have to be constructed during the process of elicitation, as an individual learns to attach numbers to feelings of uncertainty and preference, and as discussion increases the pool of knowledge and allows persuasive arguments to form and shape those feelings (Phillips, 1970; Lichtenstein and Slovic, 2006).

The emergence of PDA

This constructed view of preference is the rationale and justification for the role of decision analysis as a science of applied decision, which uses decision theoretic concepts as a basis of a practical modelling technology to *help* decision makers think more clearly and coherently about the decisions they face. This idea, which emerged through the 1960s was not significantly anticipated in the writings of Ramsey, von Neumann and Morgenstern, or Savage. Howard (1988) has described decision analysis as

> ... a systematic procedure for transforming opaque decision problems into transparent decision problems by a sequence of transparent steps ... In other words, decision analysis offers the possibility to a decision-maker of replacing confusion by clear insight into a desired course of action. (p 680)

By way of contrast, one of us has written

> Decision analysis helps to provide a structure to thinking, a language for expressing concerns of the group and a way of combining different perspectives. (Phillips, 1990, p 150)

But, however, one places the emphasis, it should be clear that decision analysis is decision theory in its work clothes: centrally concerned, not with mathematical or philosophical refinement, but with trying to be helpful.

Again, while much of the impetus came from the US (Howard, 1966; Raiffa, 1968), the idea of using decision theory as a guide to action in practical problems found a ready reception in the UK. White (1969), for example, describes his primer in the preface as the output of a research project, 'the objective of which was to ascertain the content of Decision Theory and its relevance to practical decision making', and Lindley (1971) describes his text as 'a basically non-technical book for the Administrator and Businessman'.

Over the next couple of decades, a plethora of introductory decision analysis texts appeared, intended both for business school and undergraduate curricula, and for the general business reader (Thomas, 1972; Brown *et al*, 1974a, 1974b; Moore and Thomas, 1976; Bunn, 1982, 1984; Hertz and Thomas, 1983; Smith, 1988; Morgan and Henrion, 1990; Goodwin and Wright, 1991); see also Moore's (1983) tour of a range of different application areas from a decision analytic perspective; the Anglo-American comparative review by Pearman (1987); and the snapshots of the subject (Phillips, 1989, 2005) taken by one of us for OR Society conferences. These references testify to a discipline which was becoming increasingly deeply engaged with the realities of helping decision makers with practical problems.

Alongside this growing and largely expository and pedagogical literature, a case literature started to build, starting with Beattie (1969). Two groups notable for their sustained contribution to the case literature in the early years were London-based groups of academic practitioners at London Business School (Moore *et al*, 1976) and at Brunel University, and later at the London School of Economics (eg Phillips, 1982; Phillips and Wisniewski, 1983). A striking feature of this early literature is its exclusive focus on the private sector: one of us, Phillips, recalls finding the UK civil service of this time resolutely hostile to the use of decision analysis, despite a brave attempt by Myra Chapman (Chapman, 1980) to outline its relevance to a broad range of decisions faced by British public servants (including that of whether to buy a season ticket for the daily commute from the suburbs).

Not everyone in the OR community responded to these developments with enthusiasm. KD Tocher, a former President of the OR Society, took a particular dislike to what he saw as the pretensions of this new area of OR in a paper in *JORS* in 1976:

> ... the blurb for Lindley's latest book describes it as a book about how people ought to behave. How dare anyone not behave according to the laws of my algebra! (p 234)

Tocher's critique (Tocher, 1976) and the responses which it stimulated (Adelson, 1977; Tocher, 1977a,c; French, 1978; French *et al*, 1978; Tocher, 1978) still make interesting reading today; see also French (1989) for a collection of the key articles together with a commentary by the editor; and Tocher (1977b) for an alternative presentation of Tocher's views. They stand as a reminder to one of the most critical abilities of the working decision analyst: to say, over and over again, no matter how tedious it becomes, that the aim of decision modelling is to support more thoughtful decision making rather than to automate choice. Tocher remained unpersuaded of this, maintaining in his last publication on the topic that decision analysts want to 'tell me how I ought to think ... They form a band of intellectual Mary Whitehouses who know best for all of us'. (Tocher, 1978)

Whether as a response to the challenge presented by Tocher's attack, a belated symptom of the 'crisis' in British OR of the late 1970s (Dando and Bennett, 1981; Kirby, 2000), or simply reflecting the existence of a mature community of practitioners of decision analysis, the 1980s saw the development of a literature reflecting on the craft of decision analytic modelling, dealing with issues such as the analyst/client relationship, the organizational context of analysis and differences in analysts' preferred engagement style. Thus, Ulvila and Brown (1982) describe a flexible and responsive mode of using decision analysis with busy managers; Phillips (1984) discusses the socially constructed nature of a decision analytic model and the conditions under which it can be 'requisite'; Thomas and Samson (1986) describe how decision analysis can support a 'policy dialogue' for structuring ill-structured

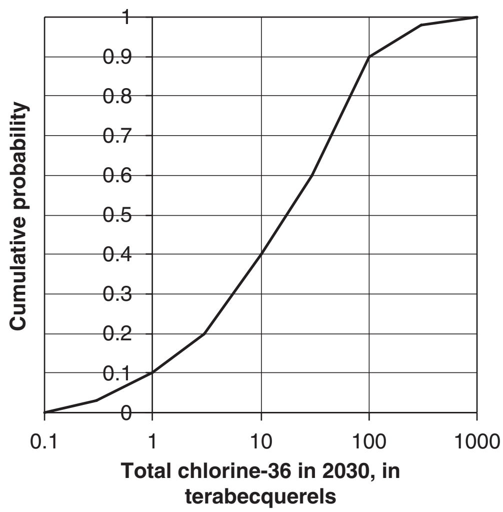

Figure 2 Judgementally assessed probability distribution for total inventory of chlorine-36 in 2030.

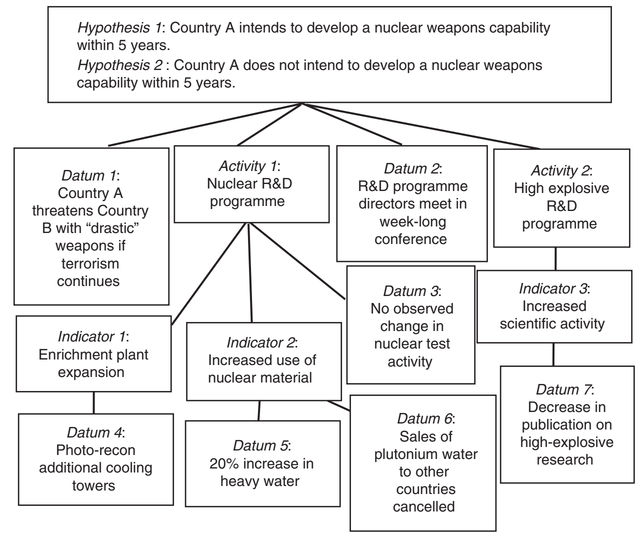

Figure 3 Bayesian network for identifying whether a country intends to develop a nuclear weapons programme. Adapted from Barclay *et al* (1977).

problems; and Watson and Buede (1987) devote a chapter of their book to the organizational aspects of decision analysis. Another notable publication of this period is French (1986), which combines an elegant presentation of the underlying mathematics of decision theory with a subtle and sensitive description of the role of decision modelling in supporting decision makers.

Although decision analysis has a case literature, it is also underpinned by a body of theory, both psychological and normative, and this too has developed considerably in the UK in the years under consideration here. The psychological theory relates primarily to the assessment of probabilities and utilities. This is a task which requires actual mental effort, since it is well-known in the psychological literature, and indeed obvious from casual introspection, people do not walk about with utility or probability density functions in their heads (and if they did there would be little call for decision analysts).

As decision analysts have increasingly preferred to elicit cardinal values directly rather than through process of equilibrating gambles (for a discussion see Goodwin and Wright, 2004, pp 119–123), we focus in this paper on the assessment of subjective probability. A landmark in the psychological treatment of this topic in the UK was the publication of two companion papers in the *Journal of the Royal Statistical Society* (Hampton *et al*, 1973; Hull *et al*, 1973). The assessment of subjective probability has become a major topic in subsequent years in the UK, with several significant books and papers by UK-based authors (Lichtenstein *et al*, 1982; Wright and Ayton, 1994; O'Hagan *et al*, 2006); and papers in Royal Statistical Society journals (eg the collection of papers in issue 1 of the 1998 volume of *The Statistician*). Such work, although of independent psychological interest, has real practical significance: by way of illustration, in Figure 2 we show an example of a probability distribution for part of the radioactive waste inventory elicited during an exercise commissioned by Nirex in the early 1990s (Phillips and Wisbey, 1993), which was one of over 40 probability distributions assessed using groups of experts. The experts constructed consensus distributions after following the Stanford Research Institute (SRI) protocol (Staël von Holstein and Matheson, 1979), adapted for use for a group of experts rather than just one individual, in a manner which characterizes the sociotechnical approach to decision analysis developed at the London School of Economics since the early 1980s.

British researchers have also played a role on the normative side of the development of decision analysis. An area where particular contributions have been made has been in the analysis of Bayesian networks. A Bayesian network is a graphical structure that depicts relationships of probabilistic dependence between a number of variables of interest. The simplest form of Bayesian network takes the form of a hierarchy or tree, which shows how the probability of an overall unobservable significant target event of interest can be related to various events which are observable, and though not significant in themselves, nevertheless have a bearing on the target event. One early text dealing (*inter alia*) with this topic is the ARPA handbook (Barclay *et al*, 1977), prepared by a team of six, three with British connections. In Figure 3, we show an example of Bayesian network from the handbook, the subject matter of which is sadly all too relevant, some three decades on. A significant step forward in both theoretic understanding and efficient computation was provided by the analysis of the conditional dependence in Bayesian networks provided by Lauritzen and Spiegelhalter (Lauritzen and Spiegelhalter, 1988; Spiegelhalter *et al*, 1993): the influence of their work can be seen in the book *Influence Diagrams, Belief Nets and Decision Analysis* (Oliver and Smith, 1990), another Anglo-American production.

PDA at 50 years out

Fifty years on from Savage's visit is a good time to take stock. In some ways, the current standing of PDA in the UK, and elsewhere, seems fairly good. As themes, 'risk' and 'risk management' enjoy considerable prominence in public discourse. Easy-to-use desktop simulation and decision analysis software (as reviewed in OR/MS Today, 2006) is easily available to managers. Excellent texts by British authors exist on the application of PDA modelling in specialized application domains, such as engineering (Bedford and Cooke, 2001), natural hazards (Woo, 1999), and health technology assessment (Briggs *et al*, 2006). Treasury guidance on investment appraisal (HM Treasury, 2003) endorses the use of explicitly probabilistic approaches, and some domain-specific UK guidance, for example on probability elicitation in radioactive waste management (Nirex, 2006), in which groups of experts construct consensus distributions, is in our view world-leading. The Stern Review on the Economics of Climate Change (Stern, 2007), one of the most prominent recent pieces of policy analysis nationally and internationally, takes uncertainty as central and addresses it probabilistically. And although the private sector is somewhat more reticent about sharing the details of decision-making practices, we are aware of plenty of activity here as well.

As these ideas become more widespread and accepted, they open up new opportunities for conducting innovative and useable research. Decision analysis applications in counter-terrorism, for example, may require drawing on game theory, and the resulting models require quite careful formal exploration before they can be solved. Elicitation throws up many research questions for both psychological and statistical communities (O'Hagan *et al*, 2006), although perhaps the biggest challenge is how to ensure that everyone who supplies input distributions for probabilistic analyses is at least aware of the many subtle ways in which their assessments can be biased. And opportunities for research on the craft aspects of decision analysis abound. For many people and organizations, explicit quantification of uncertainty and/or value is initially unfamiliar, non-intuitive, and possibly threatening; and practical use of PDA requires sensitivity to the social aspects of implementing and using decision analysis, as well as a thorough understanding of the technical aspects. Nevertheless, we detect in recent texts (Edwards *et al*, 2007), in conference presentations, and in informal conversations, a deep and developing awareness of the potential for decision analysis to be used to help individuals and organizations develop the capability to make good decisions.

Conclusion

In this paper, we have reviewed the development of PDA since Savage's visit to London in 1959, focussing on the contributions of British and UK-based workers, with a short detour to take in Ramsey's work in the 1920s. What makes Ramsey and Savage's formalisms particularly distinctive, and particularly relevant for operational researchers, is their stress on the intimate relation between uncertainty and decision, with Ramsey proposing a scheme in which the quantification of uncertainty is possible through, and acquires meaning from, its relationship with decision.

The last five decades have seen substantial progress in the transformation of the rather abstract theory of Ramsey and Savage into a practical modelling technology and a subfield of OR. We see advances on three fronts: in practical applications and the existence of a substantial case and pedagogical literature, together with documented reflections on the craft of modelling; in deeper psychological understanding of the processes by which people can be helped to construct beliefs and make probabilistic assessments; and in the development of underpinning normative theory, leading to more richer and more computationally efficient modelling tools.

This progress not withstanding, the recent run of disastrous wars and economic crises shows that our ability, as a society, to make decisions under uncertainty is far from adequate. We believe that the intellectual frameworks and modelling ideas reviewed in this paper have great potential to bring greater clarity, transparency, and accountability to these decisions. So, overall, although as explicitly probabilistic thinkers we would be the first to acknowledge that we do not know for sure what the next 50 years of PDA hold, we are optimistic about that future.

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