Interaction of defined benefit pension plans and social security

Received (in revised form): 14th February, 2008

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Abstract This paper explores the shift from defined benefit to defined contribution pension plans when the payout rate from social security is set optimally. This paper shows that when employees are receiving more of their private pensions from defined contribution plans one should be raising the payout rate from traditional social security rather than trying to privatise part of it.

Pensions (2008) **13,** 101–117. doi:10.1057/pm.2008.6

Keywords: *social security, defined benefit, defined contribution*

Introduction

Over at least the last quarter of a century, there has been a gradual transition from defined benefit to defined contribution pension plans in the United States. If anything, this trend has accelerated in the last few years. While most of this shift is occurring in the private sector, even the public sector is seeing shifts in this direction. Poterba *et al.*[1] discuss the transition in the private sector[1,2] and Fore[3] analyses changes occurring in the public sector.[3,4] Since with a defined contribution plan the return received during retirement is based on the return generated by the underlying portfolio of assets, more of the risk associated with retirement is being shifted to the retiree.[5,6]

At the same time as the above trend is occurring, there is an increased concern about the long-term viability of social security. One of the solutions being discussed is the privatisation of at least a part of social security. For a discussion of this type of privatisation see Feldstein.[7] The privatised part would have many of the attributes of a defined contribution pension plan in that the return received by the retirees would reflect the return generated by the assets held by the social security administration on behalf of the retirees. The goal of these types of proposals is to find ways of reducing the payout from the traditional social security plan, one that looks very much like the defined benefit plans that exist in the private sector.[8,9] If the overhaul of social security mentioned above is successful, that change in conjunction with the shift away from defined benefit plans will result in much more risk being borne by the average retired person.

In this paper, we look at the interaction between private pension plans and social security. More specifically, we determine the optimal level of payout by the traditional type of social security plan given the defined benefit and defined contribution pension plans provided by the

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private sector. It should not come as a big surprise that optimal social security payouts will be increased when the private sector reduces its reliance on defined benefit plans.[10,11] Moreover, to the extent that social security is not a perfect replacement for defined benefit plans and to the extent that employees are happy with their defined benefit plans, the movement towards defined contribution plans will make employees worse off.[12,13] These results are of course at odds with the general thinking about how to solve the social security insolvency issue going forward.

A related issue that is occurring at the same time is that some of the large firms that provide defined benefit plans to their workers are facing increased competition from foreign companies that are not burdened by these types of pension obligations. This foreign competition is forcing many large American corporations to downsize by laying off part of their labour force. The workers let go by these actions in many situations are forced to accept positions in new firms that have a lower salary structure and in many cases do not provide their workers with defined benefit pension plans. Even in this situation the optimal social security payout rate will be increased even though now workers will be supplying fewer funds to their defined contribution plans. The downsizing also results in a reduction in the wage structure in general and so even those individuals who retain their jobs see their salaries decline, reducing their consumption in period 1 and in turn reducing their private savings through their defined contribution plans.

In addition, in a world where both private and public pensions co-exist we analyse how changes in the dependency ratio and the degree of redistribution in social security affect the optimal level of social security payments. For example, when defined benefit plans become less generous and the dependency ratio rises, payment by social security to the lower income types will be less generous than when there is no change in the dependency ratio.[14,15] Similarly, when fewer individuals are eligible for defined benefit plans or these plans become less generous and the degree of redistribution is reduced, while the social security benefit rate and the tax on wages both rise, the payout to the poor is reduced at the same time as social security payments to the wealthy rise.

While for the basic model we assume that the magnitude of the private defined benefit plans are determined and changed exogenously, we also explore how the dependency ratio, the degree of redistribution and downsizing would affect the optimal level of both private and public pensions. We find that for all the policy changes we explored, when the payout rates of social security and the defined benefit plans are altered optimally, the two rates move in opposite directions. Thus, for example, when the number of individuals eligible for defined benefit plans is reduced the optimal payout rate on the defined benefit plans falls and the optimal payout rate on social security rises. On the other hand, when we try an experiment such as raising the dependency ratio we find that the optimal rate on social security falls but this policy at the same time raises the optimal payout rate on defined benefit plans.

Our analysis focuses on the macroeconomic implications of the change in private pension provision and thus we ignore bankruptcy issues associated with private pension plans. One of the arguments in moving away from private defined benefit pension plans is to reduce the uncertainty faced by firms and in turn to try to reduce their bankruptcy risk.[16,17] The focus in the paper is the impact of changes in pension plans on individuals. To the extent that consumers in traditional defined pension plans are protected by the Pension Benefit Guaranty Corporation, a Federal Government corporation, consumers are largely protected from the effects of bankruptcy. We, however, do ignore whatever residual risk may be present to retired individuals from these defined benefit plans.

In our model, the uncertainty considered arises from macroeconomic fluctuations. In that situation there is no riskless type of pension plan. In the case of a defined contribution plan, the risk is borne by the retiree. In the case of a defined benefit plan, even though the pensioner may view the plan as riskless, the firm, or as in our model where the burden is shifted to the employee, the worker bears the burden of the risk.[6,18]

The rest of the paper is organised as follows. The next section presents the basic theoretical model. The third section is devoted to discussing the implications of changing some of the exogenous variables. Unfortunately, for some of the results and especially when two or more exogenous variables are changed simultaneously, the results end up providing ambiguous results. Therefore the fourth section presents a numerical simulation model and discusses the lessons learned from it. This fourth section is divided into two parts. In the first part we assume that the payout rate from the private defined benefit plans is determined exogenously and in the second we assume that not only the payout rate from social security is determined optimally but also that the payout rate from private defined benefit plans is at its optimal level. The final section concludes.

The model

We use a standard overlapping generations model that has two types of individuals and two states of the world. The population is assumed to grow at the rate *n* and productivity gets enhanced at the rate *g*. All individuals work in the first period of their lives and are retired in the second. The first type of person has access to private pensions that consist of a combination of a defined benefit and a defined contribution plan. The second types are assumed to have lower incomes and do not save privately. All individuals receive benefits from a government-run social security scheme. Social security is assumed to be of the pay-as-you-go variety but we do not include a social security trust fund in the analysis.[19,20] We also allow for the possibility that the richer of the two types get back less from social security than they pay in. At any point in time there are four different groups consuming. There are the young savers, the young nonsavers, the retirees who saved privately (through their pension plans) when they were young and the retirees who rely completely on social security. The following equations specify the optimisation problem and constraints faced by type 1 individuals.

$$\max 0.5[u(c_{t1}^{1})+u(c_{t2}^{1})]+0.5\delta[u(c_{t+1,1}^{1})+u(c_{t+1,2}^{1})] \quad (1)$$

subject to:

$$c_{tj}^{1}+K_{t+1}^{1}=w_{tj}A_{tj}(1-t_{w_j}-\zeta_j)L_t^1; \quad j=1,2 \quad (2)$$

$$c_{t+1,j}^{1}=(1+r_{t+1,j}^{K})K_{t+1}^{1}+\xi\bar{w}_t L_t^1+T_{t+1}^{1}; \quad j=1,2 \quad (3)$$

$$T_{t+1}^{1}=\psi t_b \bar{w}_t L_t^1 \quad (4)$$

In equation (1) $u(.)$ is a strictly convex utility function and δ is a factor of time preference. Individuals of type 1 maximise utility over the two periods and in each period there are two possible states of the world that can each occur with probability 0.5. Individuals are represented by superscripts and subscripts are used to denote first the period and second the state of nature. The argument of the utility function is consumption, *c*. In the simulations later in the paper the utility function is assumed to take the constant relative risk aversion functional form. Equation (2) denotes the budget constraint faced by type 1 individuals of generation *t* when they are young. Type 1 individuals decide how much to consume when they are young, c_{tj}^1, $j=1,2$; and how much to save. Private saving can be done either through the defined benefit plan, where ζ_j represents the fraction of the average salary withheld from the type 1 individual and used to contribute to his defined benefit plan,[21,22] or through the purchase of private capital, K_{t+1}^1, that becomes part of his defined contribution plan. We assume that the value of K_{t+1}^1 is determined by the individual. While in some defined contribution plans the employer may be a contributer, in almost all cases the individual can supplement the firm's contribution and thus the value of K_{t+1}^1 at the margin is determined by the individual. To keep the model relatively simple we do not include bonds as one of the options for individuals in their defined contribution plan. By excluding bonds we do not have to include public goods or an income tax. In our model both the defined benefit pension plan and social security pick up some of the attributes of risk-free bonds. Moreover, for other types of bonds, capital can act as a partial proxy.

The net income of a type 1 individual when young consists of wage income from labour after taxes and after defined benefit contributions, $w_{tj}(1-t_{wj}-\zeta_j)A_{tj}L_t^1$. The amount set aside by type

1 workers when young for the defined benefit plan is assumed to be $\zeta_j w_j A_{tj} L_t^1$. Not only is the amount contributed state dependent but the percentage of the wage paid out by the young, ζ_j, is state dependent. Even though the defined benefit plan is provided by the employer, we assume that the incidence is such that the burden of the pension plan falls on the worker. The labour supply, L_t^1, can be viewed as efficiency labour and thus the amount provided can differ between type 1 and type 2 individuals. Uncertainty is introduced in this model through differences in productivity in the two states, A_{tj}, $j=1,2$. The variation in productivity in turn means that the wage rate, w_{tj}, is also state dependent. Young people pay the payroll tax that is used to finance social security, t_{wj}. In order that social security payments do not vary across the states of the world, the payroll tax rate must be state dependent. When the type 1's are old they face budget constraints (3). Their income consists of the payout from capital in the defined contribution plan, where the return on capital in period $t+1$ is $r_{t+1,j}^K$, $j=1,2$, the payout from the defined benefit plan, where ξ is the payout rate from the defined benefit plan, and the amount received from social security, T_{t+1}^1. It is assumed that the fraction ξ is a parameter and thus its value is outside the control of the individual. (In the example section we do look at some cases where the individual can control ξ and thus we will want to find the value of ξ that will maximise his utility.) The payout from the defined benefit plan is ξ times the average wage income earned in period t. We define the average wage as

$$\bar{w}_t = (w_{t1}A_{t1} + w_{t2}A_{t2})/2 \quad (5)$$

Defined benefit plans can be funded from a pool of safe assets purchased to sustain the plans. When there are, however, macro shocks, at the margin, corporations need to cover the difference between the amount paid out by the plan and the amount one can get by selling the safe assets. Thus, with uncertainty, the amount corporations need to set aside to insure that the defined benefit plans can be funded will vary. To the extent that the incidence of the tax falls on workers however, the amount that workers need to contribute will fluctuate with the uncertainty. In our simplified model, we try to address these issues by arguing that the amount workers pay in when young equals the amount paid out by the defined benefit plans to the retirees, that is,

$$\zeta_j w_{t+1,j} A_{t+1,j} L_{t+1}^1 = \xi \bar{w}_t L_t^1; \quad j=1,2 \quad (6)$$

In the steady state both the labour force and productivity grow at a constant rate and therefore the above equation can be rewritten as

$$\zeta_j w_{t,j} A_{tj} L_t^1 (1+n)(1+g) = \xi \bar{w}_t L_t^1; \quad j=1,2 \quad (7)$$

Since the payout from the defined benefit plan is state independent, we also need the relationship

$$\zeta_1 w_{t+1,1} A_{t+1,1} = \zeta_2 w_{t+1,2} A_{t+1,2} \quad (8)$$

The amount received by the type 1's from social security as denoted in equation (4) consists of the fraction ψ of the payout rate t_b times the average labour income. In this paper, we assume that social security in addition to providing a pension to all individuals does try to redistribute across income types. As will be noted below, the smaller the ψ, the larger the amount of redistribution.

Savings by the type 1's through their defined contribution plans can be found by substituting the budget constraints (2) and (3), into equation (1). Optimising over the choice variable K_{t+1}^1 yields the first-order condition:

$$-0.5[u'(c_{t1}^1) + u'(c_{t2}^1)] + 0.5\delta[u'(c_{t+1,1}^1)(1+r_{t+1,1}^K) + u'(c_{t+1,2}^1)(1+r_{t+1,2}^K)] = 0 \quad (9)$$

Type 2 individuals face a much easier problem since they do not save. Their utility function takes the form:

$$0.5[u(c_{t1}^2)+u(c_{t2}^2)]+\delta u(c_{t+1}^2) \quad (10)$$

subject to:

$$c_{tj}^2 = w_{tj}A_{tj}(1-t_{wj})L_t^2; \quad j=1,2 \quad (11)$$

$$c_{t+1}^2 = T_{i+1}^2 = \mu t_b \bar{w}_t L_t^2; \quad j=1,2 \quad (12)$$

For the type 2's, in their youth the only reduction from their gross salaries is for the payroll tax. When they are retired the type 2's rely solely on social security payments to finance consumption. Since social security payments do not vary across states, the type 2's face no uncertainty in their retirement years. The payout by social security to the type 2's is the fraction μ of the payout rate t_b times their average labour income. As noted below, the fraction μ normally takes on a value greater than 1.

Social security is of the pay-as-you-go type and so the constraint can be denoted as

$$t_{wj}w_{t+1,j}(qL_{t+1}^1+(1-q)L_{t+1}^2)A_{t+1,j} = t_b\bar{w}_t(\psi qL_t^1+\mu(1-q)L_t^2); \quad j=1,2 \quad (13)$$

Equation (13) can in turn be re-written as

$$t_{wj}w_{tj}(qL_t^1+(1-q)L_t^2)A_{tj}(1+n)(1+g) = t_b\bar{w}_t(qL_t^1+(1-q)L_t^2); \quad j=1,2 \quad (14)$$

The proportion of the type 1's in the economy is represented by q. In comparing equations (13) and (14) note that labour grows at the rate n and productivity at rate g and in the steady state $w_{tj}=w_{t+1,j}$. Finally, in order to insure that inflows to social security equal outflows we constrain ψ and μ, so that

$$\mu = 1+(1-\psi)qL_t^1/((1-q)L_t^2) \quad (15)$$

Thus, if the social security administration sets ψ, the value of μ will automatically be determined by equation (15). If social security sets Ψ to take on a value less than one, the normal case, the value of μ will take on a value greater than one. Given constraint (15), the social security budget can be simplified to yield

$$t_b\bar{w}_t = t_{wj}w_{tj}A_{tj}(1+n)(1+g) \quad (16)$$

Equation (16) holds for both states j. Note also that since social security payments do not vary with the state of nature, we have a relationship between the payroll tax rates in the two states, that is,

$$t_{w2} = t_{wt1}w_{t1}A_{t1}/(w_{t2}A_{t2}) \quad (17)$$

For the production side, we assume a standard constant returns to scale Cobb–Douglas production function that takes the form:

$$Y_{tj} = K_t^\alpha(A_{tj}L_t)^{1-\alpha}; \quad j=1,2 \quad (18)$$

where Y_{tj} represents output in period t and state j. Since the only savers in the economy are the type 1's, the aggregate capital stock at time t becomes

$$K_t = qK_t^1 \quad (19)$$

The aggregate labour supply in turn is

$$L_t = qL_t^1+(1-q)L_t^2 \quad (20)$$

where q and $1-q$ represent the proportions of the type 1's and 2's in the economy, respectively. Finally, the rates of return on the factors of production can be written as

$$\partial Y_{tj}/\partial K_{tj} = \alpha K_t^{\alpha-1}(A_{tj}L_t)^{1-\alpha} = r_{tj}^K; \quad j=1,2 \quad (21)$$

and

$$\partial Y_{tj}/\partial(A_{tj}L_t) = (1-\alpha)K_t^\alpha(A_{tj}L_t)^{-\alpha} = w_{tj}; \quad j=1,2 \quad (22)$$

The variable w_{tj} represents the wage per unit of effective labour at time t in state j and the wage per unit of L_t would be $w_{tj}A_{tj}$. It should be noted that since the growth rates are constant, we also know that $w_{tj} = w_{t+1,j}$ and $r^K_{tj} = r^K_{t+1,j}$ for $j = 1,2$.

The last part of the model involves finding the optimal values for the payroll tax rate and the social security benefit rate, t_b. The government's problem involves maximising social welfare

$$q(0.5[u(c^1_{t1}) + u(c^1_{t2})] + 0.5\delta[u(c^1_{t+1,1}) + u(c^1_{t+1,2})]) + (1-q)(0.5[u(c^2_{t1}) + u(c^2_{t2})] + \delta u(c^2_{t+1})) \qquad (23)$$

where we assume that social welfare is a weighted average of the utility of the various participants in the economy, subject to the social security budget constraint, equation (16). After substituting individual budget constraints into equation (23), the first-order condition for the government's problem after simplifying can be written as

$$qL^1_t\{-0.5[u'(c^1_{t1}) + u'(c^1_{t2})] + 0.5\delta[u'(c^1_{t+1,1}) + u'(c^1_{t+1,2})] \psi(1+n)(1+g)\} + (1-q)L^2_t\{-0.5[u'(c^2_{t1}) + u'(c^2_{t2})] + \delta u'(c^2_{t+1})\mu(1+n)(1+g)\} = 0 \qquad (24)$$

This condition is derived for the payroll tax in state 1 (although it could just as easily have been derived for the optimal social security payout rate since the two variables are related through equation (16)).

To summarise, the model described in this section consists of equations (2)–(5), (7)–(9), (11), (12), (15)–(22) and (24) in the unknowns, $c_{tj}{}^1$, $c^1_{t+1,j}$, T^1_{t+1}, $\bar{w}_t$, K^1_{t+1}, c^2_{tj}, c^2_{t+1}, μ, t_b, t_{w2}, Y_{tj}, K_t, L_t, w_{tj}, $r_{tj}{}^K$, t_{w1}, ζ_1 and ζ_2.

While for much of the paper we will analyse the model presented in this section, for some of the simulations we will assume that the defined benefit pension plan is set at its optimal level. For those simulations we need an extra equation that describes the first-order condition for the defined benefit plan, that is,

$$-0.5[u'(c^1_{t1}) + u'(c^1_{t2})]/((1+n)(1+g)) + 0.5\delta[u'(c^1_{t+1,1}) + u'(c^1_{t+1,2})] = 0 \qquad (25)$$

First-order condition (25) is derived by optimising the utility of the type 1 individuals with respect to ξ, the fraction of the average salary received by type 1's from their defined benefit plan.[23]

Implications of the theoretical model

In this subsection, we use the theoretical model specified in the previous section to analyse the impact of a reduction in the supply of defined benefit plans, a change in the degree of redistribution of social security and a change in the dependency rate. In the simulations section, we will take a closer look at interactions between these parameters as well as several other changes that yield ambiguous results when solved analytically. (An analysis of the model requires that it be differentiated and placed in matrix form. Since that exercise is largely technical in nature we relegate it to the Appendix.)

A shift away from the reliance on private defined benefit plans would have no effect on the use of defined contribution plans if there were only type 1's in the economy and social security did not redistribute. This result is due to defined benefit payments and social security benefits being perfect substitutes in our specification. One could argue that in reality these two programmes are not perfect substitutes and thus a reduction in the reliance on private defined benefit plans would increase the purchase of capital by the type 1's and in turn increase the reliance on defined contribution plans. In our model when there are both type 1's and type 2's present, even with no redistribution, a reduction in the use of defined benefit plans will result in an increase in the size of the aggregate capital stock and an increase in the use of defined contribution plans. The type 1's would like to see an increase in the social security benefit rate when the rate on defined benefit plans is reduced but the type 2's are happy with the social security rate as it is. Thus, when the defined benefit rate is reduced, while there will be an increase in the social security benefit rate, that rate will not rise as much as desired by the type 1's. Moreover, when there is redistribution, the type 1's will want there to be a smaller increase in the social security rate when the defined benefit rate is reduced and instead would want to put more of their savings into the defined contribution plans. If, before the reduction in the defined benefit pension, those employees eligible for these programmes felt that their plan was close to the optimal level from their perspective, the reduction in the size of the plan would lower their utility level at the same time as the size of the capital stock is increased. The change in the size of the capital stock does increase the wage rate but lowers the rate of return on capital.

The increase in the social security payout rate also means that the payroll tax rate will be raised as can be seen in equation (13). Thus, the after-tax income of the poor in period 1 will be reduced but their consumption in period 2 will be increased. Moreover, the greater the degree of redistribution in social security, the better off the poor will be in period 2.

A second change could be a reduction in the degree of redistribution. A reduction in the degree of redistribution means that the wealthier are better off and thus they have less need to save privately; one would therefore expect this type of change to result in a reduction in the amount of private pension money put into defined contribution pension plans and in turn in a reduction in the size of the capital stock. The lower capital stock means that the wage rate will be reduced and the rate of return on capital will be raised. The effect on the size of the payout rate from social security tends to be ambiguous. The reduction in the degree of redistribution means that the payout by social security to the poor will be reduced. Since social security is the only pension that the poor receive, they would like to see an increase in the social security pension rate even if it means higher payroll taxes on their earnings when young. On the other hand, the wealthy see their benefits from social security rise. They would therefore like to see a reduction in both the social security benefit rate and the tax rate on wages. These offsetting desires would tend to make the overall effect ambiguous; however, if the government places greater weight on the desires of the poor, one would expect both the optimal social security benefit rate and the tax on wages to rise.

The last change explored in this section is an increase in the dependency ratio. The dependency ratio can be defined as the relation between the number of people who are not working and the number of people who are. In our model, this ratio would be a relationship between the old in efficiency terms and the young in efficiency terms. (We define it in efficiency terms since both earnings when young and pensions when retired factor in the effect of productivity.) In the model this ratio can be denoted as $1/(1+n)(1+g)$. Therefore, either a reduction in the rate of labour force growth or a reduction in the rate of change of productivity will increase the dependency ratio. An increase in the dependency ratio will lower the size of the pension paid out by social security to both the poor and the wealthy. Since there are fewer young workers (in efficiency terms), less money will be collected by social security and thus there will be less money available for pensions to retirees. The wealthy react to the reduction in social security payments by increasing their contribution to their private defined contribution plans as a way to mitigate the effect of the change in social security on their retirement income. Thus, one would expect an increase in the dependency ratio to result in an increase in the size aggregate capital stock. The increase in the capital stock also means that there will be an increase in the wage rate and a reduction in the rate of return on capital. These changes also mean that consumption of both rich and poor when young will rise. Finally, the effect of an increase in the dependency rate on the benefit rate on social security and the tax rate on wages would tend to be ambiguous.

Even though this model abstracts from many aspects of reality, it is not possible to theoretically sign more complicated changes than the ones addressed in this section. To get some further insights into the issues that are the focus of this paper, the next section presents some computer simulations on the version presented in the model section. In addition, we also look at changes where both the social security benefit rate and the private defined benefit rate are determined optimally.

Results from the numerical example

This section is intended to provide further insights into some of the issues discussed in the previous part of the paper as well as to explore some related topics. In the numerical example, we use a social welfare function and individual utility functions that are assumed to be isoelastic. The utility functions take the form:

$$U^j(.) = 0.5[(c^j_{t1})^{(1-\gamma_j)} + (c^j_{t2})^{(1-\gamma_j)}]/(1-\gamma_j) + 0.5\delta[(c^j_{t+1,1})^{(1-\gamma_j)} + (c^j_{t+1,2})^{(1-\gamma_j)}]/(1-\gamma_j); \quad j = 1,2, \gamma_j \geqslant 0, \tag{26}$$

where γ_j is the Arrow-Pratt measure of relative risk aversion.

In addition, for the simulations we use the somewhat more general CES production function that takes the form:

$$Y_{tj} = [(a_K K_t)^\rho + (a_L A_{tj} L_t)^\rho]^{1/\rho}; \quad j = 1,2 \tag{27}$$

where a_K is the coefficient on capital in the CES production function, a_L is the coefficient on labour in the CES function, and ρ takes on values from 1 to $-\infty$.

The simulation model in this section does not use parameter values that pretend to mimic the American or any other existing economy. Rather, the policy changes introduced here are intended to indicate the types of effects one would expect to occur in an economy from the policy alterations that are analysed in the example. Moreover, the policy changes introduced are relatively large so that one can easily see the various impacts. One should also note that we have collapsed the many years in the lives of workers and retirees into two periods. This section is divided into two subsections. The first analyses policy changes when the social security tax rate and hence the social security benefit rate are set at their optimal levels but the private defined benefit rate is exogenous, and the second discusses the effects of policy variations when both the social security benefit rate and the private defined benefit rate are set at their optimal levels.

Defined benefit rate set exogenously

The simulations described in this section are summarised in Table 1. The first row of the table describes the base model. In the base model, the exponent of the utility of consumption of the type 1's is 3 and of the type 2's is 4. We assume that the rate of labour force growth, n, and the rate of technological change, g, are each 80 per cent. The labour supply of type 1 individuals, L_t^1, takes a value of 30 and of the type 2's, L_t^2, is 10. The discount factor, δ, takes on a value of 0.9. We assume that the proportion of type 1's is 0.7. In the base model, we assume redistribution and so the type 1's get back a quarter of what they put in, that is, $\psi = 0.25$. The level of productivity in state 1, A_{t1}, is 2 and in state 2, A_{t2}, is 0.5. In the CES production the coefficient on capital takes on a value of 0.333 and that of labour is set at 0.666, and ρ has a value of 0.6. Finally, the benefit rate on private defined benefit pensions, ξ, is 0.17775. The rate ξ was set at the optimal level given the initial conditions so that the base case for both Tables 1 and 2 would be the same. We did try other parameter values for the base model and found that the results below were robust to changes in the initial parameter values.

In Table 1 we summarise the values of the most important variables. The ones that are left out either do not add much to the explanation, or, as in the case of the utility of the participants, the direction of change can actually be inferred by seeing how individual consumptions move. Moreover, in those cases where first and second period consumption move in opposite directions, what happens to the value of utility is strongly dependent on the magnitude of the exponent of consumption in the utility function, a parameter for which it is difficult to come up with a 'true value'.

The first policy change in Table 1 replicates the exercise of the theoretical section by lowering the benefit rate on defined benefits from 0.17775 to 0.12775. As before, this change results in an increase in the capital stock and the wage rate. As noted earlier, a policy change that lowers the

Table 1: Numerical example where defined benefit rate is set exogenously

Policy	*Change*	c^1_{t1}	c^1_{t2}	$c^1_{t+1,1}$	$c^1_{t+1,2}$	c^2_{t1}	c^2_{t2}	c^2_{t+1}	T^1_{t+1}	K^1_{t+1}	t_b	ω_{t1}	$\bar{\omega}_{t1}$
		36.84	6.399	12.891	10.56	13.379	3.23	5.94	0.71	1.88	0.1105	0.684	0.8599
ξ=0.12775		36.91	6.408	12.896	10.25	13.423	3.26	5.98	0.72	2.34	0.1108	0.686	0.8635
ξ=0.17775	ψ=0.5	36.80	6.412	12.973	10.93	13.241	3.11	5.70	1.90	1.51	0.1478	0.682	0.8567
ξ=0.12775	ψ=0.5	36.89	6.444	12.967	10.60	13.281	3.13	5.78	1.93	1.93	0.1493	0.684	0.8603
ξ=0.17775	*q*=0.6	36.63	6.226	12.706	10.46	13.241	3.11	5.65	0.97	1.68	0.1507	0.682	0.8572
q=0.6	ψ=0.5	36.59	6.252	12.856	10.97	13.084	2.97	5.34	2.46	1.26	0.1925	0.680	0.8535
n=0.7	*g*=0.7	36.72	6.251	12.457	10.27	13.376	3.22	5.66	0.68	1.82	0.1052	0.685	0.8612
n, *g*=0.7	ξ=0.12775	36.82	6.289	12.507	10.00	13.425	3.25	5.72	0.69	2.30	0.1057	0.687	0.8653
n, *g*=0.7	*q*=0.6	36.48	6.059	12.238	10.15	13.227	3.09	5.39	0.92	1.62	0.1435	0.683	0.8583

Table 2: Numerical example where defined benefit rate is set endogenously

Policy	*Change*	c^1_{t1}	c^1_{t2}	$c^1_{t+1,1}$	$c^1_{t+1,2}$	c^2_{t1}	c^2_{t2}	c^2_{t+1}	T^1_{t+1}	K^1_{t+1}	t_b	ω_{t1}	$\bar{\omega}_t$	ξ
		36.84	6.399	12.891	10.56	13.38	3.23	5.94	0.71	1.880	0.1105	0.684	0.8599	0.17775
ψ=0.5		36.89	6.442	12.967	10.64	13.28	3.13	5.77	1.92	1.882	0.1492	0.684	0.8599	0.1335
q=0.6		36.68	6.243	12.713	10.25	13.27	3.13	5.68	0.97	1.977	0.1511	0.684	0.8596	0.1440
n=0.7	*g*=0.7	36.86	6.298	12.531	9.88	13.45	3.26	5.74	0.69	2.54	0.1059	0.688	0.8672	0.1042
n, *g*=0.7	*q*=0.6	36.69	6.133	12.365	9.56	13.33	3.14	5.49	0.94	2.67	0.1448	0.688	0.8568	0.0689

payout rate on defined pension plans induces individuals to increase their savings through defined contribution pensions. In addition, for the type 1's there will be an increase in consumption in period 1 and an ambiguous impact on consumption during retirement. (Consumption rises in state 1 and falls in state 2.) Moreover, there is an increase in the pensions paid out by social security to the two types and a slight increase in the social security payout rate. The type 2's see an increase in consumption in both periods. Not surprisingly, the type 1's are made slightly worse off by this change and the type 2's are made better off.

The second exercise is to reduce the amount of redistribution by providing the better off with 50 per cent of the social security benefit rate instead of the 25 per cent they received in line 1 of Table 1. This change increases the payoff of social security to the well off. This increase induces them to reduce the amount they save through their defined contribution plans resulting in the amount of capital they buy falling from 1.88 to 1.51. The reduction in the capital stock also means that the wage rate is reduced. These changes mean that the poor consume less in both the first and second period. The pressure by the poor to increase their social security payout also means that the benefit rate for social security will rise from 0.1105 to 0.1478. This change results in an ambiguous impact on consumption for the type 1's in period 1, reducing consumption in state 1 and increasing it in state 2, and an increase in their consumption in period 2. For the type 1's, the increase in the pension they receive from social security is larger than the reduction in income from their defined contribution plans. By increasing the payout to the type 1's their utility is increased and by reducing the payout to the type 2's the poorer types have their utility reduced. If there were only type 1's in the economy, one would expect t_b to be reduced to partially offset the effect of the higher ψ. The type 2's, however, see their social security payments reduced by the policy change and thus to partially mitigate that reduction they want t_b raised.

The next exercise is a combination of a change in the defined benefit rate and in the degree of redistribution. Thus, the change is from $\xi = 0.17775$ and $\psi = 0.25$ to $\xi = 0.12775$ and $\psi = 0.5$. The combination of policy changes raises the social security benefit rate from 0.1105 to 0.1493. This policy places opposing pressures on the size of the capital stock. The net effect, however, is to increase the capital stock from 1.88 to 1.93 as well as to increase the average wage rate, from 0.8599 to 0.8603. Also, the social security payments to the type 1's increases from

0.71 to 1.93 which is, in fact, a larger increase than if there had been only a decrease in the amount of redistribution. For the type 2's this policy change results in a decrease in consumption in both periods but for the type 1's consumption rises in both periods.

Thus far we have looked at cases of reductions in defined benefit plans where the decrease stems from a lower defined benefit rate. While some firms have been making their defined benefit plans less generous, others have kept the benefit rate on their plans unchanged but rather have been forced to retrench. Thus, some firms have been forced to lay off workers, perhaps due to foreign competition. The workers laid off in many cases are forced to take positions in firms that are much less generous than the ones they left. Thus these workers are forced to leave older established firms that had defined benefit pension programmes. The new firms that hire these workers tend to be younger and may have benefit programmes that are less generous in general but more specifically are likely only to have defined contribution pension plans and not defined benefit pension plans. In our model, one can get an idea of the impact of such changes by lowering the value of q. In the next exercise we lower the value of q from 0.7 to 0.6 but keep all the other parameters at the same value as in the base model. The lower q results in a decrease in the number of individuals saving privately as well as a reduction in the amount each type 1 puts into his/her defined contribution plan. The reduction in the capital stock also means that the average wage in the economy falls. These changes mean that the consumption of the type 1's and the type 2's fall in both period 1 and 2. The reduction in q also means that the fraction of the payout from social security received by the type 2's is reduced as can be seen in equation (15). The lower average wage and lower μ mean that the poor would like to see the benefit rate on social security, t_b, raised. These pressures mean that the optimal rate on social security rises from 0.1105 to 0.1507. Nonetheless, the other two effects dominate and so the pension received by the type 2's fall. On the other hand, the social security payments received by the type 1's rise but that increase is not large enough to offset the effect of the lower capital and the lower average wage and so the consumption of the type 1's in period 2 falls. In fact, this change results in the consumption of both types falling in both periods and thus welfare being reduced for both of them as well.

In comparing the two ways in which defined benefit plans can become less of a factor in the economy, one immediately notices that when firms reduce the benefit rate, ξ becomes smaller, the size of the capital stock and the wage rate rise, whereas when the number of workers who are eligible for defined benefit pensions is reduced, both the capital stock and the wage rate are reduced. Moreover, when ξ is reduced consumption tends to rise but when q falls consumption for the two types falls in both periods. In addition, social security payments to the poor rise when ξ falls, but are reduced when q is lowered. On the other hand, social security payments to the type 1's rise in both cases.

The next example explored is to combine a reduction in the number of individuals eligible for defined benefit plans with a change in the degree of redistribution. Here we reduce q from 0.7 to 0.6 and raise ψ from 0.25 to 0.5. In this case the capital stock and wages fall even more than when only q is reduced. The combined change reduces the capital stock from 1.88 to 1.26 whereas it only falls to 1.68 when only q is reduced. In addition, the increase in the benefit rate on social security and the tax rate on wages rise more with the combined change. The increase of t_b is from 0.1105 to 0.1925 with the combined change and only to 0.1507 when q alone changes. The poor are, however, nonetheless made worse off by the combined change. Their consumption falls further in both periods. On the other hand, the type 1's see their consumption rise when they are retired with the combined change but their consumption is affected ambiguously in period 1. The social security payment to the type 1's is higher with the combined change but wages are lower and the tax on wages is higher when both q and ψ change. One should note, however, that consumption for the type 1's in period 1 is lower than for the base case but changes ambiguously in period 2.

The next issue explored in this model is an increase in the dependency ratio. As noted in the theoretical section, the higher dependency ratio can be a result of a reduction in the labour force growth rate, a lower rate of technological change or a combination of the two. Here we look at the implications of lowering both g and n from 0.8 to 0.7. Note that our model collapses many periods in an individual's life into two. The higher dependency ratio results in a reduction in the social security rate, t_b, as well as lower social security payments to the retirees. The type 1's will also decrease the amount they place in defined contribution plans and thus one will get a decrease in the capital stock, although the average wage actually rises. In addition, the higher dependency ratio will result in the tax rate on wages going up. The above changes mean that consumption for both types falls in both periods.

The last examples in this section are ones where we combine an increase in the dependency ratio with a decrease in defined benefit plans. The reduction in the emphasis on defined benefit plans is occurring at the same time as there is heightened concern about an increase in the dependency ratio. The concern about the dependency ratio stems not only from reduced birth rates and therefore lower rates of population growth but also increases in longevity. While our model is not rich enough to allow for changes in life expectancy, changes in population growth rates and rates of technological change can act as proxies for changes in longevity.

In the first case, we increase the dependency ratio by reducing the growth rate of labour and the rate of technological change from 0.8 to 0.7 and also reduce the payout rate from the defined benefit plans from 0.17775 to 0.12775. In comparing a decrease in the defined benefit rate in an economy without a change in the dependency ratio with the one where there is an increase, one finds that the capital stock rises less, the average wage increases more and the consumption of the type 1's will be smaller than those when there is no change in the dependency ratio. For example, the capital stock rises from a base value of 1.880 to 2.30 when there is a change in the dependency ratio instead of to 2.34 when there is no change, and consumption of the type 1's in period 1 will be 36.82 in state 1 when the dependency ratio changes whereas it will be 36.91 when there is no change in that ratio. On the other hand, payments by social security will be smaller when there is a rise in the dependency ratio. For example, the payment by social security to the poor retirees rises from 5.94 to 5.98 when there is no change in the dependency ratio, but actually falls to 5.72 when the dependency ratio increases. Finally, when the dependency ratio rises the social security benefit rate falls, although the tax rate on wages did rise further with the higher dependency ratio.

In the last example, we reduce the number of workers eligible for defined benefit plans and compare economies where the above reduction occurs in an economy where there has been no change in the dependency ratio and in one where the ratio rises. In this case the value of q is reduced from 0.7 to 0.6 and both the rate of population growth and the rate of technological change are reduced from 0.8 to 0.7. When the number of people eligible for defined benefits falls, both the value of capital and the average wage are reduced. When the dependency ratio rises, the decrease in the value of capital is even larger, although the change in the average wage is smaller. For example, capital falls from 1.88 to 1.62 when the dependency rises rather than to 1.68 when there is no change. Also the social security benefit rate rises less when the dependency ratio increases, going from 0.1105 to 0.1435 rather than to 0.1507 when there is no change in the dependency ratio; however, the tax rate on wages rises further, going from 0.0214 to 0.0312 in state 1 as opposed to rising to 0.0292 when there is no change in the ratio. In addition, the payout to the poor from social security falls further when the dependency ratio goes up, declining from 5.94 to 5.39 as opposed to 5.65 when there is no change and consumption by the type 1's also falls more in their second period, going, for example in state 1, from 12.891 to 12.238 instead of to 12.706.

Thus, the increase in the dependency ratio in both cases causes the social security benefit rate to be reduced even more than when there is no change in the dependency rate and the payments by social security to both types falls even further than when that rate does not change.

Defined benefit rate set optimally

In this section of the paper we assume that both the social security benefit rate (and of course, the tax rate on wages) and the defined benefit rate are set at their optimal levels. To obtain the optimal values for these variables, we use the first-order condition found in equation (25) where the utility function, as in the previous section, takes the isoelastic form. The simulations here look at the effects of changes in the degree of redistribution, of alterations in the number of individuals eligible for defined benefit plans and of an increase in the dependency ratio on the payout rates from both social security and defined benefit plans. The results of the simulations are summarised in Table 2. The first row of Table 2 summarises the values for the base model. The base model here uses the same initial conditions as the one used in the previous part.

The first experiment is to reduce the degree of redistribution. As before, we change the value of ψ from 0.25 to 0.5. Raising ψ to 0.5 ends up reducing the optimal payout rate on the defined benefit plans from 0.17775 to 0.1335 but at the same time raising the payout rate from social security from 0.1105 to 0.1492 and the tax rate on wages in state 1 from 0.0214 to 0.0290. This policy ends up pretty much leaving the capital stock and the average wage unchanged. One should notice from the previous section that the increase in ψ reduces the size of the capital stock and the average wage and the lower benefit rate on defined benefit plans raises them. When we raise ψ and allow ξ to adjust optimally however, the two effects pretty much cancel each other out. This policy not surprisingly raises the payout from social security to the type 1's and in turn raises their retirement consumption (as well as their period one consumption). On the other hand, the consumption for the type 2's falls in both periods. Not surprisingly, the utility of the rich is raised and the welfare of the poor is reduced by this policy. From the perspective of the type 1's, they view social security and the defined benefit plans as somewhat substitutable. Since the type 2's, because of the higher ψ and thus the lower payout by social security to them, want t_b raised, the type 1's desire a lower value for ξ.

The second policy is the one that reduces the number of individuals eligible for defined benefit plans at the same time as the defined benefit rate adjusts optimally. The policy change that reduces q from 0.7 to 0.6 results in an increase in the amount added by the type 1's to their defined contribution plans, the amount going from 1.88 to 1.977. Nonetheless, overall there is a decrease in the size of the aggregate capital stock and the average wage rate because of the reduction in the number of type 1's in the economy. Moreover, while the optimal payout rate from the defined benefit plans falls from 0.17775 to 0.1440, the optimal rate from social security rises from 0.1105 to 0.1511 as does the tax rate on wages from 0.0214 to 0.0293. The above changes mean that the consumption of both types fall in both periods and the welfare of both types is reduced. One should note that consumption for the type 1's falls when they are retired even though their social security payments rise.

The third policy change is the one where the dependency ratio is raised by reducing n and g from 0.8 to 0.7 at the same time as the rate on the defined benefit plans adjusts endogenously to its optimal level. This policy lowers the payout rate on social security from 0.1105 to 0.1059, and also reduces the optimal payout rate on the defined benefit plans from 0.17775 to 0.1042. These changes mean that the type 1's increase the amount they save through their defined contribution plan by increasing the amount they invest in capital from 1.880 to 2.54. These changes mean that both the size of the aggregate capital stock and the average wage are raised. The net effect of this policy is to lower consumption of both types when they are retired but it has an ambiguous effect on their consumption when young. Given our utility function, this policy lowers utility for both types.

The last case is the one that combines the last two policies above. Now we raise the dependency rate by lowering n and g from 0.8 to 0.7, lower the number of individuals eligible for defined benefit plans from 0.7 to 0.6 and allow the rate on defined benefit plans to adjust optimally. That policy increases the amount of capital bought by the type 1's from 1.88 in the standard model to 2.67 with the new policy. Also, the average wage rises from 0.8599 to 0.8668. In addition, the policy raises the payout rate on social security from 0.1105 to 0.1448 but lowers the optimal rate on defined benefit plans from 0.17775 to 0.0689. In terms of consumption, both types see a reduction in their consumption in both periods. This policy, unfortunately, also has the effect of reducing the level of utility for both types.

In summarising the impact of the policy adjustments in this section, the most important result that we may have obtained is that all of the policy adjustments cause the optimal payout rates from social security and from the defined benefit plans to move in opposite directions. Thus, when a policy causes the optimal payout rate on the defined benefit plans to be reduced, that policy also induces the optimal payout rate on social security to rise and vice versa.

Conclusion

One of the most striking changes in the private sector over the last quarter of a century has been the transition from defined benefit to defined contribution pension plans. While this shift has been widespread among private pension plans, we have also seen both discussion and implementation of individual or personal accounts in the public sector. These types of accounts can range from defined contribution accounts managed for the public sector by private managers to notional accounts.

In this paper, we study the distributive incidence of such a shift in private pensions. The cornerstone of our argument is perhaps the naïve assumption, namely that the government keeps the same objective function before and after the shift and uses social security for offsetting the incidence of such a move.

Within such a setting, we show that the optimal social security system responds to the shift from defined benefit pensions to defined contribution pension plans. As one would expect, the optimal social security payout will be increased when the private sector reduces its reliance on defined benefit plans. Two cases are analysed. In the first one, the defined benefit rate is set exogenously; in the second, it is set optimally. In this latter case, the shift is expressed by reducing the number of retirees eligible for defined benefits private plans. The main conclusion of our paper is that any shift towards defined contribution calls for an offsetting reform of social security. This is not surprising. In a country such as Sweden that is attached to redistribution, the introduction of notional defined contribution plans in public pensions has been accompanied by the development of a means-tested guaranteed pension. Along the same lines, if a country such as the United States maintains the same social welfare objectives as those prevailing at the start of social security, one would expect that the shift towards defined contribution pensions would be accompanied by an increase in the payout ratio of social security.

The fact that we do not observe such offsetting changes in social security means that at the same time as the private sector is shifting more of the uncertainty about retirement from the workers to the retirees, the government is making similar types of decisions. If, in our model, the government were to reduce the weights it places on retirees as compared to workers at the same time as the private sector is shifting to a greater emphasis on defined contribution plans, we would have found a reduced emphasis on the 'traditional' type of social security offerings and more emphasis on both public and private defined contribution type plans.

Thus, one conclusion to be drawn from our analysis is that governments are signalling that the earlier attempts to guarantee the good life to retirees by placing much of the risk and burden on the workers has gone too far. The new view is that there must be more of a balance in how one treats workers relative to retirees and hence more of the risk associated with retirement in the future will need to be borne by the retirees themselves.

References

1 Poterba, J. M., Venti, S. F. and Wise, D. A. (2004) 'The transition to personal accounts and increasing retirement wealth: Macro- and microevidence', in Wise, D.A. (ed.) 'Perspectives on the Economics of Aging, A National Bureau of Economic Research Conference Report', The University of Chicago Press, Chicago and London, pp. 17–79.

2 'The transition from employer managed defined benefit pensions to retirement saving plans that are largely managed and controlled by employees has been the most striking change in retirement saving over the last two decades. Individual managed and controlled retirement accounts, particularly 401(k) plans but also 403(b) plans for nonprofit organisations, 457 plans for state and local employees, the Thrift Savings Plan for federal employees, Keogh plans for self-employed workers, and Individual Retirement Accounts (IRAs), have grown enormously. Employer-provided defined benefit pensions have declined in importance. In 1980, 92 per cent of private retirement saving contributions were to employer-based plans; 64 per cent of these contributions were to DB plans. In 1999, about 85 per cent of private contributions were to accounts in which individuals controlled how much to contribute to the plan, how to invest plan assets and how and when to withdraw money from the plans' (Poterba *et al.*,[1] pp. 17–18).

3 Fore, D. (2001) 'Going private in the public sector: The transition from defined benefit to defined contribution pension plans', in Mitchell, O. S. and Hustead, E. C. (eds.) 'Pensions in the Public Sector', University of Pennsylvania Press, Philadelphia, pp. 267–287.

4 'There is evidence that the public sector pension environment is beginning to evolve. A small but growing number of state and local governments have switched or are contemplating switching from a defined benefit to a defined contribution plan. If these pioneers prove successful, in terms of employee and employer satisfaction, the public sector may follow the transition trend experienced in the private sector over the last quarter century' (Fore,[3] p. 267).

5 Bodie *et al.*[6] discuss some of the trade-offs when comparing defined benefit and defined contribution plans.

6 Bodie, Z., Marcus, A. J. and Merton, R. C. (1988) 'Defined benefit versus defined contribution pension plans: What are the real trade-offs?', in Bodie, Z., Marcus, A. J. and Merton, R. C. (eds.) 'Pensions in the U.S. Economy, A National Bureau of Economics Research Project Report', The University of Chicago Press, Chicago and London, pp. 139–160.

7 Feldstein, M. (2005) 'Structural reform of social security', *Journal of Economic Perspectives*, Vol. 19, pp. 33–55.

8 For defined benefit plans 'employees are guaranteed a pension of a given amount per year upon retirement. The benefit level is often a function of one or more factors, such as an employee's years of service and earnings profile over his or her tenure with the firm' (Barnow and Ehrenberg,[9] p. 524.) However, the earnings profile used in defined benefit plans is usually much shorter than that used for social security.

9 Barnow, B. S. and Ehrenberg, R. G. (1979) 'The cost of defined benefit pension plans and firm adjustments', *Quarterly Journal of Economics*, Vol. 93, pp. 523–540.

10 In the past, many private defined benefit plans were integrated with social security. For a discussion, see Bodie *et al.*[11] However, as we move away from defined benefit plans this integration is becoming less prevalent.

11 Bodie, Z., Marcus, A. J. and Merton, R. C. (1987) 'Pension plan integration as insurance against social security risk', in Bodie, Z., Shoven, J. B. and Wise, D. A. (eds.) 'Issues in Pension Economics, A National Bureau of Economics Research Project Report', The University of Chicago Press, Chicago and London, pp. 147–169.

12 In looking at the trade-off between defined benefit plans and social security, one of the disadvantages of social security from the point of view of higher paid workers is that it is redistributive, but one of the advantages is that the benefits are indexed for inflation whereas most defined benefit plans provide a constant retirement benefit fixed in nominal terms. For a discussion of the inflation factor associated with defined benefit plans see Bodie.[13]

13 Bodie, Z. (1990) 'Pensions as retirement insurance', *Journal of Economics Literature*, Vol. 28, pp. 28–49.

14 Apps and Rees argue that 'an ageing population is associated with an increasing ADR (aged dependency ratio), but it will also be associated with a declining child dependency ratio (CDR), when the root cause of population ageing is declining fertility. It does not then necessarily follow that the net effect of the fertility decline must be a reduction in the share of aggregate output consumed by the working population' (Apps and Rees,[15] p. 573).

15 Apps, P. and Rees, R. (2002) 'Fertility, dependency and social security', *Australian Journal of Labour Economics*, Vol. 5, pp. 569–585.

16 In terms of funding there is a big difference between defined contribution and defined benefit plans. 'In defined contribution plans, by definition the value of the benefits equal that of the assets, so the plan is always exactly fully funded. But in defined benefit plans, there is a continuum of possibilities. There may be no separate fund, in which case the plan is said to be unfunded. When there is a separate fund with assets worth less than the present value of the promised benefits, the plan is underfunded. And if the plan's assets have a market value that exceeds the present value of the plan's liabilities, it is overfunded' (Bodie and Papke,[17] p. 152).

17 Bodie, Z. and Papke, L. E. (1992) 'Pension fund finance', in Bodie, Z. and Munnell, A. H. (eds.) 'Pensions and the Economy, Pension Research Council Publications', Pension Research Council and University of Pennsylvania Press, Philadelphia, pp. 149–172.

18 'The pegging of benefits in DB plans to final average wage would appear to provide employees with a type of income-maintenance insurance not available in DC plans. This observation has been used to support the selection of these plans over DC plans. This conclusion, however, is not robust. If wage paths are unpredictable at the start of a career, then individuals may view it as very risky to have their retirement benefits depend so heavily on final salary' (Bodie *et al.*,[6] p.147).

19 For a discussion of the implications of having a trust fund one can look at Pestieau and Possen.[20]

20 Pestieau, P. and Possen, U. M. (2000) 'Investing social security in the equity market. Does it make a difference', *National Tax Journal*, Vol. 53, pp. 41–57.

21 In this paper, we are not attempting to do a general equilibrium analysis of defined benefit plans. Ebrahim[22] argues that if one wanted to do a general equilibrium analysis one would need to define them in terms of 'not only the income but also expected portfolio payoffs. This is radically different from that observed in the 'real world', where it is defined strictly in terms of pensionable salary, which is derived only using the average of

that in pre-retirement years (after incorporating the number of years of service)' (see Ebrahim,[22] p. 15).

22 Ebrahim, M. S. (2006) 'Pension fund design and corporate structure: A general equilibrium exposition', unpublished.

23 Given our specification, if there were only type 1's in our model and no redistribution, defined benefits and social security would be perfect substitutes. In general, however, one would not expect such an outcome since social security and defined benefit plans use different earnings profiles to determine benefits and unlike social security most defined benefit plans do not index benefits for inflation after retirement.

Appendix

If one substitutes equations (2)–(5), (7), (8), (10)–(12), (15)–(18), (19) and (20) into equations (9), (21), (22) and (24) and differentiates the resultant model, one can express the results as

$$\begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ B_1 & 0 & -1 & 0 \\ C_1 & -1 & 0 & 0 \\ D_1 & D_2 & D_3 & D_4 \end{bmatrix} \begin{bmatrix} \mathrm{d}K^1_{t+1} \\ \mathrm{d}w_{t1} \\ \mathrm{d}r^K_{t+1,1} \\ \mathrm{d}t_{w1} \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}$$

where

$$A_1 = 0.5\left\{ \begin{array}{c} [u''(c^1_{t1}) + u''(c^1_{t2})] \\ +\delta[u''(c^1_{t+1,1})(1+r^K_{t+1,1})^2 + u''(c^1_{t+1,2})(1+r^K_{t+1,2})^2] \end{array} \right\} < 0$$

$$A_2 = 0.5\left\{ \begin{array}{c} -[u''(c^1_{t1})(1-t_{w1}-\zeta_1) + u''(c^1_{t2})[(A_{t1}/A_{t2})^{\alpha-1} - (t_{w1}+\zeta_1)]] \\ A_{t1}L^1_t + \delta[u''(c^1_{t+1,1})(1+r^K_{t+1,1}) + u''(c^1_{t+1,2})(1+r^K_{t+1,2})] \\ \{(\zeta_1 + \psi t_{w1})A_{t1}L^1_t(1+n)(1+g)\} \end{array} \right\}$$

$$A_3 = 0.5\delta\left[\begin{array}{c} u''(c^1_{t+1,1})(1+r^K_{t+1,1})K_{t+1} + u'(c^1_{t+1,1}) + u''(c^1_{t+1,2}) \\ (A_{t2}/A_{t1})^{1-\alpha}(1+r^K_{t+1,2})K_{t+1} + u'(c_{t+1,2})(A_{t2}/A_{t1})^{1-\alpha} \end{array} \right]$$

$$A_4 = 0.5\left\{ \begin{array}{c} [u''(c^1_{t1}) + u''(c^1_{t2})]A_{t1}L^1_t w_{t1} \\ +\delta[u''(c^1_{t+1,1})(1+r^K_{t+1,1}) + u''(c^1_{t+1,2})(1+r^K_{t+1,2})] \\ \psi A_{t1} w_{t1} L^1_t (1+n)(1+g) \end{array} \right\} < 0$$

$$B_1 = \alpha(\alpha-1)(K_{t+1})^{\alpha-2} q (A_{t1}L^1_t)^{1-\alpha}(1+n)^{1-\alpha}(1+g)^{1-\alpha} < 0$$

$$C_1 = \alpha(1-\alpha)(A_{t1}L^1_t(1+n)(1+g))^{-\alpha}(K_{t+1})^{\alpha-1} q > 0$$

$$D_1 = 0.5qL_t^1 \left\{ \begin{array}{c} [u''(c_{t1}^1) + u''(c_{t2}^1)] \\ +\delta \begin{bmatrix} u''(c_{t+1,1}^1)(1+r_{t+1,1}^K) \\ +u''(c_{t+1,2}^1)(1+r_{t+1,2}^K) \end{bmatrix} \psi(1+n)(1+g) \end{array} \right\} < 0$$

$$D_2 = 0.5qL_t^1 \left\{ \begin{array}{c} -\begin{bmatrix} u''(c_{t1}^1)(1-t_{w1}-\zeta_1) + u''(c_{t2}^1) \\ [(A_{t1}/A_{t2})^{\alpha-1} - t_{w1} - \zeta_1] \end{bmatrix} A_{t1}L_t^1 \\ +\delta[u''(c_{t+1,1}^1) + u''(c_{t+1,2}^1)] \\ (\psi t_{w1} + \zeta_1)A_{t1}L_t^1(1+n)(1+g) \\ \psi(1+n)(1+g) \end{array} \right\}$$
$$+(1-q)L_t^2 \left\{ \begin{array}{c} -0.5[u''(c_{t1}^2)(1-t_{w1}) + u''(c_{t2}^2)((A_{t1}/A_{t2})^{\alpha-1} - t_{w1})] \\ A_{t1}L_t^2 + \delta u''(c_{t+1}^2)\mu^2 \\ t_{w1}A_{t1}L_t^2(1+n)^2(1+g)^2) \end{array} \right\}$$

$$D_3 = 0.5q\delta[u''(c_{t+1,1}^1) + u''(c_{t+1,2}^1)(A_{t2}A_{t1})^{1-\alpha}]$$
$$K_{t+1}^1\psi(1+n)(1+g)L_t^1 < 0$$

$$D_4 = 0.5qL_t^1 \left\{ \begin{array}{c} [u''(c_{t1}^1) + u''(c_{t2}^1)]A_{t1}L_t^1 w_{t1} \\ +\delta[u''(c_{t+1,1}^1) + u''(c_{t+1,2}^1)]\psi^2 A_{t1}w_{t1}L_t^1(1+n)^2(1+g)^2 \end{array} \right\}$$
$$+(1-q)L_t^2 \left\{ \begin{array}{c} 0.5[u''(c_{t1}^2) + u''(c_{t2}^2)]w_{t1}A_{t1}L_t^2 \\ +\delta u''(c_{t+1}^2)\mu^2 w_{t1} \\ A_{t1}L_t^2(1+n)^2(1+g)^2 \end{array} \right\} < 0$$

$$E_1 = -0.5 \left\{ \begin{array}{c} ([u''(c_{t1}^1) + u''(c_{t2}^1)]/((1+n)(1+g)) \\ +\delta[u''(c_{t+1,1}^1)(1+r_{t+1,1}^K) + u''(c_{t+1,2}^1)(1+r_{t+1,2}^K)])\bar{w}_t L_t^1 \end{array} \right\} \mathrm{d}\xi$$
$$-0.5\delta \left\{ \begin{array}{c} [u''(c_{t+1,1}^1)(1+r_{t+1,1}^K) + u''(c_{t+1,2}^1)(1+r_{t+1,2}^K)] \\ t_{w1}w_{t1}(1+n)(1+g)A_{t1}L_t^1 \end{array} \right\} \mathrm{d}\psi$$
$$-0.5\delta\{[u''(c_{t+1,1}^1)(1+r_{t+1,1}^K) + u''(c_{t+1,2}^1)(1+r_{t+1,2}^K)]w_{t1}A_{t1}L_t^1\}$$
$$(\psi t_{w1} + \zeta_1)\{(1+g)\mathrm{d}n + (1+n)\mathrm{d}g\}$$

$$E_2 = \alpha(\alpha-1)(K_{t+1})^{\alpha-1}(A_{t1}L_t^1)^{1-\alpha}$$
$$\{(1+n)^{-\alpha}(1+g)^{1-\alpha}\mathrm{d}n + (1+n)^{1-\alpha}(1+g)^{-\alpha}\mathrm{d}g\}$$

$$E_3 = \alpha(1-\alpha)(K_{t+1})^{\alpha}(A_{t1}L_t^1)^{-\alpha}$$
$$\{(1+n)^{-\alpha}(1+g)^{-\alpha-1}\mathrm{d}g + (1+n)^{-\alpha-1}(1+g)^{-\alpha}\mathrm{d}n\}$$

$$E_4 = -0.5qL_t^1 \left\{ \begin{array}{c} ([u''(c_{t1}^1) + u''(c_{t2}^1)]/((1+n)(1+g)) \\ +\delta[u''(c_{t+1,1}^1) + u''(c_{t+1,2}^1)]\psi(1+n)(1+g))\overline{w}_t L_t^1 \end{array} \right\} \mathrm{d}\xi$$
$$-qL_t^1\delta[t_{w1}A_{t1}w_{t1}(1+n)^2(1+g)^2$$
$$\{[u''(c_{t+1,1}^1/2) + u''(c_{t+1,2})/2]\psi L_t^1 - u''(c_{t+1}^2)\mu L_t^2\}$$
$$+\{u'(c_{t+1,1}^1) + u'(c_{t+1,2}^1) - u'(c_{t+1}^2)\}(1+n)(1+g)]\mathrm{d}\psi$$
$$-\left\{ \begin{array}{l} 0.5qL_t^1\delta \left(\begin{array}{c} [u''(c_{t+1,1}^1) + u''(c_{t+1,2}^1)]\psi(\psi t_{w1} + \zeta_1)w_{t1}A_{t1}L_t^1 \\ (1+n)(1+g) + [u'(c_{t+1,1}^1) + u'(c_{t+1,2}^1)]\psi \end{array} \right) \\ +0.5(1-q)L_t^2\delta \left[\begin{array}{c} u'(c_{t+1}^2)\mu \\ +A_{t1}L_t^2 t_{w1} w_{t1}\mu^2(1+n)(1+g)u''(c_{t+1}^2) \end{array} \right] \end{array} \right\}$$
$$\{(1+g)\mathrm{d}n + (1+n)\mathrm{d}g\}$$

One condition that is assumed to hold and that helps to insure that the results discussed in the paper go through is

$$\begin{array}{l} u''(c_{t+1,1}^1)[(1+r_{t+1,1}^K) - (1+n)(1+g)] \\ +u''(c_{t+1,2}^K)[(1+r_{t+1,2}^K) - (1+n)(1+g)] < 0 \end{array}$$

We found that this condition is likely to be satisfied. In general, whether this condition is satisfied or not depends on the third derivative of the welfare function. When the condition is satisfied, the determinant of the model is negative. Moreover, one can verify that the comparative static results discussed in the theoretical section will go through.