ADDITIONAL PAPER

EMSR versus EMSU: Revenue or utility?

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ABSTRACT:
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An assumption of risk-neutrality lies at the heart of the standard algorithm for leg inventory optimisation (EMSR). This paper examines how to modify EMSR scientifically to account for different risk preferences (eg risk aversion). This new approach is called expected marginal seat utility (EMSU). A comparison is made between the differences in booking limits that are generated by the different risk preferences. Simulation analysis is used to show what the differences in expected revenues and expected utilities are between the two methods. The results show that switching to EMSU in cases of risk aversion can increase expected utility by 1.6–4 per cent.

INTRODUCTION

Theoretical work in the area of airline seat inventory control started in the early 1970s with approaches to finding seat allocations on a single flight leg for two classes based on equating marginal unit revenues. Beginning with Littlewood (1972), several researchers published very similar approaches to determining the number of seats to be made available to 'full fare' as opposed to 'discount fare' passengers. See Belobaba (1987), Weatherford (1991), Weatherford and Bodily (1992) or McGill and van Ryzin (1999) for more comprehensive overviews of the earliest works dealing with what was at that time limited to the two-class seat allocation problem, on a single leg or multiple legs.

More relevant to the development of current revenue management (RM) systems is the introduction, beginning in the 1980s, of more than two different price types, all of which share the entire inventory of the cabin on a single leg, and the concurrent modification of reservations control systems to allow multiple nested booking classes. Belobaba (1989) first developed a heuristic decision rule for finding seat protection levels and booking limits for more than two nested booking classes, a model that has come to be known as the EMSR (expected marginal seat revenue) model. The optimal set of conditions for determining the protection levels and nested booking limits in multiple price class structures were subsequently presented by Curry (1990), in an approach commonly known as OBL (optimal booking limits).

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Similar and independently derived optimal solutions to the same multiple nested class problem were also published by Brumelle *et al*. (1990) and Wollmer (1992).

Belobaba (1992) then developed a modified version of his original EMSR seat protection model, more closely approximating the characteristics of the optimal conditions for nested booking classes. This revised model is now known as the EMSRb heuristic model, the details of which were also published by Belobaba and Weatherford (1996). The original version of Belobaba's model is now referred to as the EMSRa heuristic. In the EMSRb model, joint protection levels are calculated for all higher classes relative to a given lower class, based on a combined demand forecast and a weighted average price level for all classes above the one for which a booking limit is being calculated. The weighting is done based on expected demand to come in that class.

Most airline, hotel, rental car and cruise line RM systems currently in use today utilise EMSRa, EMSRb or some variant thereof in determining booking limits for multiple nested booking classes on a single leg. Expected marginal seat revenue has been used for over a decade as the airline industry standard for leg seat inventory control.

The author was asked by a consulting client (Greg Estep at Open Skies by Navitaire) who worked with many smaller airlines whether EMSR was still the best approach for leg inventory control for an airline that was risk averse. After a little thought, it became apparent that the answer was a resounding 'no!'. There is a whole body of literature, called utility theory, dealing with different risk preferences. The literature was surveyed to see whether this important concept of utility theory had been previously addressed in the context of EMSR, and it was found that not one scholarly article had been written on the topic. The rest of this paper will be organised as follows: the standard EMSRb model for seat protection is reviewed with a sample calculation, the new utility model approach (EMSU) is presented, and then a comparison between the revenue and utility performance of the two decision rules is provided. Finally, some conclusions are presented.

This paper again focuses on the single leg and presents empirical findings from simulation experiments in which these two different leg RM strategies are compared (EMSR versus EMSU). The simulation analysis is limited to a comparison of 'optimisation' models for a *single leg* with multiple booking classes, which is the level at which the vast majority of airlines run their RM systems today.

STANDARD MODEL FOR SEAT PROTECTION — EMSRb

The following modelling assumptions about the nature of fare class demand and the booking process are made for EMSRb under the condition of serially nested classes:

(1) Demand for each fare class is separate and independent of demand in other classes.
(2) Demand for each fare class is stochastic and can be represented by a probability distribution.
(3) Lowest class books first, in its entirety, followed by the next lowest class, etc.
(4) Spilled passengers (ie ones that are turned down on a given flight) are lost to the company completely (ie not accommodated on another flight).
(5) All demands arrive in a single booking period (ie static optimisation model).
(6) The company is risk-neutral (ie the company is indifferent between a sure \$100 and a 50 per cent chance of \$200 (with a corresponding 50 per cent chance of \$0) — a new key assumption which has, until now, been overlooked.

Table 1: Example of random demand and fares

Fare class	*Avg. demand*	*Std. dev.*	*Fare ($)*
Y	40	10	500
B	50	15	300
M	60	20	100

For a simple example calculation, consider the following flight leg with three fare classes and random demand and fares as in Table 1.

Applying the standard EMSRb calculations and assuming the aircraft had capacity of 150 seats, the booking limits would be

$$BL_{\mathrm{Y}} = 150$$
$$BL_{\mathrm{B}} = 150 - 37 = 113$$
$$BL_{\mathrm{M}} = 150 - 101 = 49$$

(see Belobaba (1989) or Weatherford (2002) for details on how these numbers were derived).

The next section shows how these booking limits change as assumptions about a company's risk preference change.

NEW UTILITY MODEL — EMSU

What if the airline company is not comfortable with assumption (6) above? For example, suppose a smaller company is not willing to take as many risks? Perhaps a larger company is experiencing a cash flow struggle and would rather have the sure cash (low fare) now, than some chance of getting more cash (higher fare) later? That is, instead of being risk-neutral, a company is actually risk averse. It is situations like these where the new concept of EMSU can be of help. In practice, similar risk-averse behaviour by airline analysts can be observed: a few days before departure, analysts start to confirm waiting lists, because they are not comfortable with the recommendation of their RM system to wait for a high-fare passenger that might (or might not) appear. The new concept introduced here is that, instead of expected revenue being maximised, expected utility will be maximised. Utility is a way of measuring how much the revenue means to the company. See Lindley (1985) for a good reference on utility theory as applied to general decision making.

On a personal level, readers may have already experienced (or at least can imagine) that earning the first $1,000 meant a lot more when they were coming out as poor, college graduates, than it did to receive $1,000 once they were more established (eg the utility of the first $1,000 is much higher than going from $199,000 to $200,000).

So, the first step is to determine what the company's risk preference is. There are several ways to do this, but one relatively simple way is to compare the following situations:

— *Situation 1*: a 50–50 chance of winning either $100 or $0
— *Situation 2*: a certain cash pay-off of $$x$.

The company must then answer the question: how big would x have to be to make it indifferent between the two situations? Of course, the answer could be different for each company (and for each individual). If the company decides that x would have to be exactly $50, then the company is risk neutral. If the company picks a value for x that is less than $50 (eg $40), then the company is risk averse. Obviously, the lower the value for x, the more risk averse the company is. Finally, if the company picks a value for x that is more than $50, the company would be risk seeking. This behaviour is not typical for companies, but this behaviour is observed in numerous individuals (eg those who like to go to Las Vegas and gamble). In the practice of RM by airlines, the choice between situation 1 and situation 2 above is not just made once a day, but thousands of times a day. So the real choice is more like

— *Situation 1*: The sum of 1,000 independent experiments, each with a 50–50 chance of winning either \$100 or \$0
— *Situation 2*: A certain cash pay-off of $1{,}000 \times \$x$.

The simple demonstration above is not sufficient to help a company choose an entire risk profile. Once the company has established which of the three broad categories it falls into (risk averse, risk neutral, risk seeking), the next step is to calculate the utility for all monetary amounts that the company is likely to face. It is assumed in the remainder of this paper that the company is risk averse. If it were risk neutral, standard EMSRb would be the proper algorithm to use. If the company were risk seeking, the following steps could be easily modified to match that characteristic. To make the utility calculation, one of the easiest ways to convert from a dollar (\$) amount to a utility is to use a standard exponential curve of the form

$$U(x) = 1 - \exp(-x/riskconstant)$$

where $U(x)$ is the utility of $\$x$, and exp() is base e raised to the given power.

Choosing different values for the risk constant affects the shape of the curve, as shown in Figure 1. In this example, with the maximum fare equal to \$500 and the lowest outcome of \$0, the utility anchor points are defined as $U(\$0) = 0$ and $U(\$500) = 1$.

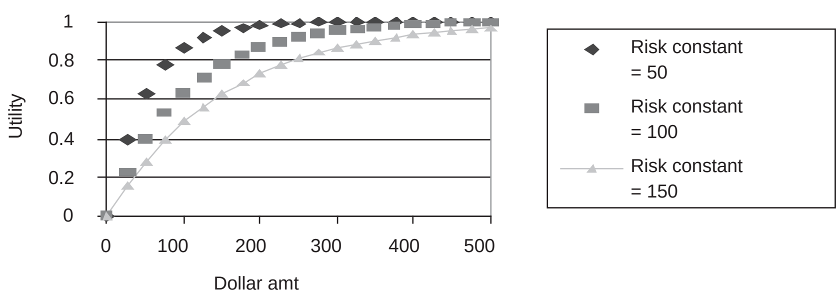
The larger the risk constant, the more the utility curve starts to look like a line with slope equal to 1.0 (ie represents risk-neutral behaviour). The smaller the risk constant, the greater the curvature and the more risk-averse the behaviour. In practice, the best way to choose the risk constant would be to simulate several different options and present them to the company for final choice.

The standard EMSRb calculation (see Belobaba (1989) or Weatherford (2002)) is modified as follows to incorporate the utility of the revenue (fare) being earned. The expected marginal utility of making the Sth seat available to class i is defined as follows

$$EMSU_i(S_i) = U(R_i) \times P_i(S_i)$$

where $U(R_i)$ is the utility of the revenue (or fare) from class i, and $P_i(S_i)$ is the probability of selling greater than or equal to S_i

Figure 1: Utility values for varying dollar amounts (for different levels of risk aversion)



seats to class i. The optimal protection level π_{YB} for class Y from class B satisfies

$$EMSU(\pi_{YB}) = U(R_B)$$
$$U(R_Y) \times P(\pi_{YB}) = U(R_B)$$

As is done with standard EMSR, once π_{YB} is found, $BL_B = \text{capacity} - \pi_{YB}$ is set. Of course, $BL_Y = \text{capacity}$.

Now getting back to the specific example, in order to find the protection for the Y fare class, it is neceassry to find the largest value of π_{YB} for which:

$$EMSU(\pi_Y) = U(R_Y) \times P(\pi_{YB}) \geqslant U(R_B)$$

Assuming the risk constant equals \$50, then

$$EMSU(\pi_{YB})$$
$$= U(\$500) \times P(\pi_{YB}) \geqslant U(\$300)$$
$$= 0.999955 \times P(\pi_{YB}) \geqslant 0.997521$$
$$P(\pi_{YB}) \geqslant 0.99757$$

where $P(\pi_{YB})$ is the probability that random demand for class $Y \geqslant \pi_{YB}$. If it is assumed that demand in Y class is *normally* distributed with a mean and standard deviation given earlier, then a standard normal table can be used to determine that $\pi_{YB} = 11$ is the largest integer value of π_{YB} which gives a probability greater than or equal to 0.99757 (that random demand in Y class will be at least 11 seats), and therefore 11 seats will be protected for Y class.

Doing a similar calculation for the Y&B classes together to determine the protection level from class M, one can determine that 70 seats should be protected for combined classes Y&B. Suppose the aircraft has capacity of 150 seats, then the booking limits would be

$$BL_Y = 150$$
$$BL_B = 150 - 11 = 139$$
$$BL_M = 150 - 70 = 80$$

As one can observe by comparing the utility-based booking limits above with the standard EMSR booking limits in the second section, these EMSU seat allocation decisions are much more conservative (ie more risk averse) in that they protect many fewer seats for the upper classes and allow more to be sold to the more 'sure' lower fare classes.

COMPARISON OF THE PERFORMANCE OF THE EMSR AND EMSU DECISION RULES

Visual C++ was used to code up the simulation and, for each simulation scenario, 50,000 trials were run to ensure statistically significant revenue differences. An interesting question to ask is what impact does this new EMSU decision rule and its associated booking limits have on the revenue and utility generated? With the three fare class example and a plane with a capacity of 150 (data already shown earlier), the difference in booking limits generated when the risk constant equals \$50 has already been seen.

Example 1: Risk constant = \$50

The impact on utility and revenue was as follows: EMSR generated an average utility of 126.02, while EMSU generated an average utility of 131.56, for a 4.4 per cent increase by EMSU.

The average numbers that were booked in each class by the two different decision rules are shown in Table 2.

The EMSR algorithm generated an average load factor of 88.1 per cent, while the EMSU algorithm had an average load factor of 92.8 per cent. The yield (average fare/passenger) was \$291.89 for the EMSR algorithm, while EMSU had a yield of \$275.94. Average revenue was \$38,615 for EMSR and \$38,231 for EMSU, for a 1.0 per cent increase by EMSR. It can be clearly seen that EMSR holds back more seats in Y, taking the chance that they will be filled at higher fare (result = lower load factor, higher yield), while EMSU takes on less risk and lets more be sold in the M class.

Table 2: Average numbers booked in each class by two different decision rules

	EMSR	*EMSU*	*Difference (EMSR–EMSU)*
Y	39.3	37.5	+1.8
B	48.4	46.5	+1.9
M	44.5	55.1	–10.6

One may wonder why EMSU should ever be used, since it does not maximise revenue, and utility cannot be spent. The reason is that EMSU maximises the probability of hitting certain revenue thresholds. For example, in this case, the probability that EMSU generates at least \$35k in revenue is 77.9 per cent, whereas EMSR only hits the same threshold 74.4 per cent of the time.

Example 2: Risk constant = \$150

If the risk constant is now increased to \$150, meaning that the company is less risk averse than in Example 1, the following booking limits are obtained

$BL_Y = 150$
$BL_B = 150 - 28 = 122$
$BL_M = 150 - 89 = 61$

These booking limits generate the following impact on revenue and utility: EMSR generated an average utility of 101.36, while EMSU generated an average utility of 102.99, for a 1.6 per cent increase by EMSU.

The average numbers that were booked in each class by the two different decision rules are shown in Table 3.

The EMSR algorithm generated an average load factor of 88.1 per cent, while the EMSU algorithm had an average load factor of 91.6 per cent. The yield (average fare/passenger) was \$291.77 for the EMSR algorithm, while EMSU had a yield of \$280.92. Average revenue was \$38,604 for EMSR and \$38,504 for EMSU, for a 0.26 per cent increase by EMSR. Again, it can be seen clearly that EMSR holds back more seats in Y, taking the chance that they will be filled at a higher fare (result = lower load factor, higher yield), while EMSU takes on less risk and lets more be sold in the M class although, compared with Example 1, the risk aversion is less and so the percentage improvement in average utility by EMSU is less, as is the decrease in expected revenue. In this example, the probability that EMSU generates at least \$35k in revenue is 77.6 per cent, whereas EMSR only hits the same threshold 73.9 per cent of the time.

Table 3: Average numbers booked in each class by two different decision rules

	EMSR	*EMSU*	*Difference (EMSR–EMSU)*
Y	39.3	38.3	+1.0
B	48.3	47.3	+1.0
M	44.6	51.7	–7.1

CONCLUSIONS

In the evolution of RM theory and practice, no attention has been paid to the risk preference of companies as incorporated into the optimisation models used to allocate units to different fare classes. This paper has focused on the single leg seat optimisation problem, exploring the impact on expected revenue performance simulated under varying risk preferences and has compared the performance of the most commonly used EMSR decision model to a new decision rule (EMSU). A booking simulation was applied to evaluate the revenue and utility performance of the new EMSU decision rule versus the standard EMSR decision rule.

The simulation results showed that incorporating actual risk preference which is different from risk neutrality (eg risk aversion) can have a significant impact on the expected utility and revenue performance. In fact, utility improvements of 4 per cent on legs where the risk constant equals $50 and utility improvements of 1.6 per cent on legs where the risk constant equals $150 (less risk aversion) have been demonstrated. Even though one cannot spend utility, this paper has shown that EMSU generates a greater probability of hitting certain revenue thresholds than does EMSR.

Some areas for further study may include: (a) exploring what risk profiles are adopted by firms in the airline industry as well as other industries and how similar/different they are; (b) how real companies choose their risk profile; and (c) running simulations with various risk profiles and surveying practitioners at companies of various sizes and states of financial health about their preferences for the profiles.

One of the nice features of this new decision rule is that it does not require a different control mechanism from the traditional availability displays of the CRS. The results of these simulations should not be interpreted as conclusive with respect to the exact utility and revenue impacts that can be expected for every company, as it was seen that differences in the levels of risk aversion can cause substantially different impacts. These findings do demonstrate, however, the importance of the new decision rule introduced in the literature via this paper: expected marginal seat utility.

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